Math 140

Exam on Chapter 5 Information

What you should know:

- Really pay attention to the general power rule for integrals. It only works when \( n \neq -1 \):
  \[
  \int x^n \, dx = \frac{x^{n+1}}{n+1} + C, \quad n \neq -1.
  \]

- We define \( \ln(x) = \int_1^x \frac{1}{t} \, dt, \quad x > 0 \)

- You should know the properties of the natural log function listed on page 319, Theorem 5.1.
- You should know the log properties listed on page 319, Theorem 5.2.
  \[
  \ln(e) = \int_1^e \frac{1}{t} \, dt = 1
  \]

- \( \frac{d}{dx} (\ln(x)) = \frac{1}{x}, \quad x > 0 \). You should also know the Chain Rule version of this.

- You should know how to use the log rules to expand log expressions before taking derivatives.
- You should know how to use logarithmic differentiation on complicated, non-log expressions. See example 6 on page 323.
  \[
  \int \frac{1}{x} \, dx = \ln|x| + C. \quad \text{Do NOT forget the absolute values!}
  \]

- You should know the strategy to determining how to evaluate an integral:
  1. Can you integrate using a known formula? If so, use the formula.
  2. Can you use algebra or trig to simplify the integrand and then integrate using a known formula?
  3. Try a \( u \)-substitution. Make sure you find \( du = f'(x)dx \) and solve for \( dx \).
    a. If there is a composition, try to let \( u = \) “inner function” of the composition.
    b. If there is not a composition or part a fails, try to let \( u = \) “more complicated” function

- The integration formulas you need to know, listed in the front cover of your book, are: 1 through 10, 12, 14 through 19.

- You need to know the basics about inverse functions: \( f(f^{-1}(x)) = x \) and \( f^{-1}(f(x)) = x \), the domain and range of \( f(x) \) and \( f^{-1}(x) \) are switched, a inverse function is unique, \( f(x) \) and \( f^{-1}(x) \) are symmetric about the line \( y = x \), how to determine if a given function has an inverse, how to find an inverse function.

- Do not memorize Theorem 5.9, the derivative of an inverse function on page 341, but do know how to use it if given the theorem.

- You need to know that \( y = \ln(x) \) and \( y = e^x \) are inverse functions.

- Be able to switch from exponential form to log form and vice versa.
- How to solve a log equation.
- How to solve an exponential equation.

- Review your exponent rules! Know that \( e^a \cdot e^b = e^{a+b} \) and \( \frac{e^a}{e^b} = e^{a-b} \)

- You should know the properties of \( y = e^x \), see the box at the bottom of page 347.
• Know that \( \frac{d}{dx} (e^x) = e^x \) and the Chain Rule form.

• Know that \( \int e^x \, dx = e^x + C \)

• Know how to work with bases other than \( e \) in logs or exponentials.

• It is really helpful to know the change of base formula for logs: \( \log_b x = \frac{\ln x}{\ln b} \)

• It is also really helpful to know how to re-write any exponential function with base \( e \): \( a^x = e^{(\ln a)x}, \ a \neq 1 \)

• You should be able to differentiate logs and exponentials with bases other than \( e \), and also apply the Chain Rule as needed. \( \frac{d}{dx} (a^x) = \frac{d}{dx} (e^{(\ln a)x}) = (\ln a) a^x \) and \( \frac{dy}{dx} (\log_b x) = \frac{dy}{dx} \left( \frac{\ln x}{\ln b} \right) = \frac{1}{(\ln b)x} \).

• You need to be able to distinguish between expressions that are exponential and those that “look exponential.” See example 5 on page 359. I did something similar in class during the lecture for section 5.5.

• You should be able to use the compounding interest formulas on page 360. Don’t memorize them.

• You need to know the domain and range for the 3 following inverse trig functions: \( y = \sin^{-1}(x), \ y = \cos^{-1}(x) \) and \( y = \tan^{-1}(x) \).

• You should be able to evaluate inverse trig functions without a calculator, solve an inverse trig equation, and use right triangles to solve for the values of inverse trig functions.

• You need to know the derivatives of \( y = \sin^{-1}(x), \ y = \cos^{-1}(x) \) and \( y = \tan^{-1}(x) \).

• You need to know the integrals involving inverse trig functions. See Theorem 5.17, parts 1 and 2, on page 375.

• You need to know the definitions of the hyperbolic sine and cosine functions (see page 383), as well as know their graphs, domains and ranges (see page 384).

• You need to know the 3 hyperbolic identities listed below.
  - \( \cosh^2 x - \sinh^2 x = 1 \)
  - \( \tanh^2 x + \sech^2 x = 1 \)
  - \( \sinh(2x) = 2 \sinh(x) \cosh(x) \)

• You need to know the 6 derivative and integral formulas for hyperbolic functions listed below.
  - \( \frac{d}{dx} (\sinh x) = \cosh(x) \quad \int \cosh(x) \, dx = \sinh(x) + C \)
  - \( \frac{d}{dx} (\cosh x) = \sinh(x) \quad \int \sinh(x) \, dx = \cosh(x) + C \)
  - \( \frac{d}{dx} (\tanh x) = \sech^2(x) \quad \int \sech^2(x) \, dx = \tanh(x) + C \)

• You DO NOT need to memorize the formulas for the inverse hyperbolic functions but you do need to know how to use them! The formulas are listed on page 387 in your textbook, Theorem 5.19.

• You DO NOT need to memorize the formulas for the derivatives and antiderivatives of the inverse hyperbolic functions but you do need to know how to use them! The formulas are listed on page 389 in your textbook, Theorem 5.20.

**Review Problems**

On pages 393 through 394 in your book is the Chapter 5 Review. I suggest you be able to do the following problems.

Do problems: 1-93 odds.