Lab: Approximating Area with the Trapezoidal Rule

Competency Problem:

You are working for a small business that manufactures ultralight aircraft in Temecula. The owner likes to experiment with design modifications and is presently trying different shaped wings on some of his aircraft. He knows that you are taking calculus and asks you to calculate the area of the wing whose outline is shown in Figure 1 (next page). Use the trapezoidal rule to calculate the area. Use enough subdivisions to give a good estimate. Accuracy will improve as the number of subdivisions increases.

1. Make a table showing the measurements you used and turn it in with your report.
2. Area of the wing using the trapezoidal rule (include units): _________________
3. Explain why your answer seems reasonable. Contrast your answer with what you might expect to obtain by using one or two rectangles as a rough estimate.

Do not work on the competency problem until you have completed the problems below.

Purpose: The purpose of this lab is to learn how to use the trapezoidal rule to find the area of a region in the plane. **Do not use definite integrals to solve the problems on this lab!**

Lab Checklist: The following items are due.

1. The guided problem where you will estimate an area using the trapezoidal rule.
2. The answer to the Competency Problem involving the aircraft wing above.
3. All graphs must have the axes clearly labeled and scaled appropriately.
4. All tables must be filled in.
5. All questions must be answered.

Notes:

1. Do NOT solve this competency problem until you have finished the following guided problems.
2. You may use technology to such as your graphing calculator, Wolfram Alpha, etc.Either print out and turn in your work, or copy it down on this paper.
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Name:

Discussion: We can calculate the area of a rectangular region in the plane by multiplying its length by the width. We also have formulas for calculating the area of triangles, trapezoids, circles and certain other well-known geometric shapes. We can also use Calculus to develop formulas for certain other regions in the plane. Most irregular regions require approximation methods. Many of these approximation methods involve dividing (partitioning) the interval on the $x$-axis into smaller sub-intervals and then approximating the area on each sub-interval using a geometric shape that can be easily computed (in this case, trapezoids). The total area is then found by the sum of the areas on the sub-intervals.

The area of a single trapezoid is given by the formula: 

$$A = \frac{1}{2}(b_1 + b_2)h$$

If $f(x)$ is a continuous function on $[a, b]$ and we use a regular partition of $[a, b]$, where $\Delta x = \frac{b-a}{n}$, then the area of the $i$th trapezoid is given by:

$$A_i = \frac{1}{2}(f(x_{i-1}) + f(x_i)) \Delta x = \left(\frac{f(x_{i-1}) + f(x_i)}{2}\right) \left(\frac{b-a}{n}\right)$$

The sum of the areas of the $n$ trapezoids is given by:

$$\left(\frac{f(x_0) + f(x_1)}{2} + \frac{f(x_2) + f(x_3)}{2} + \cdots + \frac{f(x_{n-1}) + f(x_n)}{2}\right) \left(\frac{b-a}{n}\right) = \frac{b-a}{2n} \left(f(x_0) + 2f(x_1) + \cdots + 2f(x_{n-1}) + f(x_n)\right)$$

Guided Problem:

Problem 1: Use the trapezoidal rule to estimate the area between $f(x) = \frac{\cos(x)}{x} + 1$ and $g(x) = 0$ from $x = 0.1$ to $x = 2.5$ with $n = 6$.

a. Graph $f(x)$ over the restricted domain $[0.1, 2.5]$. 

![Graph of f(x) over the restricted domain [0.1, 2.5].](image)
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b. If $n$ represents the number of rectangles required, what is $n$ for this problem?

c. Recall that $\Delta x = \frac{b-a}{n}$. What is $\Delta x$ for this problem? $\Delta x = ?$ ______________

d. Complete the following table. Round the function values to 4 decimal places.

<table>
<thead>
<tr>
<th>$i$</th>
<th>$x_i$</th>
<th>$f(x_i)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
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<td>2</td>
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<tr>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
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</tbody>
</table>

e. Compute the area using the Trapezoidal Rule formula.

$$\text{Area} = \text{_________________________}$$

f. Draw in the 6 trapezoids on the graph that you used to approximate the area.

Back to the Competency Problem:

Use the techniques learned in this lab to estimate the area of the aircraft wing. Use enough rectangles to give a good estimate.

- Show your work on separate paper and turn it in with this lab. Show your work for full credit.
- Turn in your final table that you used to estimate the area. Make sure your answer is VERY PRECISE and includes UNITS.
- Number of trapezoids used: ________________________________
- $\Delta x = \text{________________} \text{ and the area is } \text{________________} \text{ (units).}$