Math 140  Logistic Growth Models  Section 6.3

We have discussed the limitations of the exponential growth model \( \frac{dy}{dx} = ky \), which has solution \( y = Ce^{kt} \). This models populations which exhibit unlimited growth. Even in the case of bacteria, eventually the population would run out of space, or food, or kill the host.

In a logistic growth model, we consider the growth of a population which grows rapidly (exponentially) at first, but has an upper limit on the population size imposed by environmental factors. This upper limit is called \( L \), sometimes known as the carrying capacity. The carrying capacity is the maximum population size that can be sustained as times goes on.

The logistic growth model is: \( \frac{dy}{dt} = ky(L - y) \quad 0 < y < L \quad \text{for } t \geq 0 \)

Here \( L \) is the carrying capacity and \( y \) is the population at time \( t \). This equation says the population \( y \) is increasing at a rate that is jointly proportional to its size \( y \) and the difference between \( L \) and its size \( y \).

To solve this equation, first separate the variables.

\[
\frac{1}{y(L - y)} \, dy = k \, dt
\]

Integrate both sides:

\[
\int \frac{1}{y(L - y)} \, dy = \int k \, dt
\]

Write the left side as a sum of partial fractions.
Integrate both sides.

Solve for $y$.

Let’s sketch a graph of this curve. Let $y = f(t)$ . Now, find $y = f(0)$ .