Jan. 28, 2015

- Quiz on 1.4, 1.5

Feb. 2
- Next Monday
- Exam on it
- All of Chapter 1
- Current exam
- Review Sec. 1.5
- Start Sec. 2.1

Today
- Today

Feb. 5
- Finish up next exam
- Review for Exam 1
- Review for Exam 1

Review for Exam 1
sec. 1.5 (cont.)

review

- Limits at infinity
- Vertical asymptotes
- Hole in graph vs. V.A.?

ex: \( f(x) = \frac{x - 6}{x^2 - 3x} \)

\[
f(x) = \frac{x - 6}{(x + 6)(x - 6)}
\]

- \( x = 6 \):
  \( f(6) = \frac{6 - 6}{(6+6)(6-6)} = \frac{0}{0} \) - removable discontinuity

- \( x = -6 \):
  \( f(-6) = \frac{-6 - 6}{(-6)^2 - 3(-6)} = \frac{-12}{0} \) - non-removable discontinuity.
(new in 1.5)

\[ \lim_{x \to -6^-} \left( \frac{1}{x+6} \right) = -\infty \]

\[ \begin{align*}
X &= -7 \\
X &= -6.5 \\
X &= -6.1 \\
X &= -6.01
\end{align*} \]

**Ex:** Find the vertical asymptotes

\[ f(x) = \frac{x^2 - 4}{x^3 + 2x^2 + x + 2} \]

Where is \( \text{denom} = 0? \)

4 terms - try grouping

Group 1st 2 terms

\[ \text{Group 2nd 2 terms} \]

\[ \frac{x^3 + 2x^2 + x + 2}{0} \]

or

\[ (x^3 + 2x^2) + (x+2) = 0 \]

\[ x^2(x+2) + 1(x+2) = 0 \]
real solutions only!

\[(x+2)(x^2+1) = 0\]

\[x + 2 = 0 \quad x^2 + 1 = 0\]

no real solutions.

\[x = -2\]

\[f(x) = \frac{x^2 - 4}{x^3 + 2x^2 + x + 2} = \frac{(x-2)(x+2)}{(x+2)(x^2+1)}\]

removable discontinuity at \(x = -2\)

no v.a.

ex: Find v.a. of \(g(\theta) = \frac{\tan \theta}{\theta}\)

\(\theta = 0\) is not domain

\[g(0) = \tan(0) = 0\]

hole in graph at \(\theta = 0\) even though calc does not show it.
\[
\lim_{\theta \to 0} \frac{\sin \theta}{\cos \theta} = 1
\]

Properties of Infinite Limits:

Let \( f, g : \mathbb{R} \to \mathbb{R} \) be functions such that \( \lim_{x \to c} f(x) = \infty \) and \( \lim_{x \to c} g(x) = L \).

\[
\lim_{x \to c} [f(x) + g(x)] = \infty
\]

Thm 1.1.5

Let \( c \in \mathbb{R} \).

\[
\lim_{x \to c} \frac{1}{f(x)}
\]
\[ f(x) = \begin{cases} 
0 & \text{if } x < 0 \\
-\infty & \text{if } x = 0 \\
-8 & \text{if } x > 0
\end{cases} \]

\[ g(x) = \begin{cases} 
8 & \text{if } x < 0 \\
-\infty & \text{if } x = 0 \\
8 & \text{if } x > 0
\end{cases} \]

\[ \text{Line } C \times C \]

\[ \lim_{x \to -\infty} \left( \frac{g(x)}{f(x)} \right) = \frac{-\infty}{0} = -\infty \]

\[ \lim_{x \to 0^+} \left( \frac{g(x)}{f(x)} \right) = \frac{-\infty}{0} = -\infty \]

\[ \lim_{x \to 0^-} \left( \frac{g(x)}{f(x)} \right) = \frac{-\infty}{0} = -\infty \]

A statement like \( \infty - \infty \) is undefined. Why is this ok? 

ex: \( \lim_{x \to 0} -\frac{1}{x^3} = -\infty \)
Our goal: Find the equation of the tangent line to the curve \( y = f(x) \) at a point \( x = c \).

Tangent (tan.) line vs. Secant line:
- Tangent line touches curve near \( x = c \) only at \( x = c \).
- Secant line touches curve at 2 pts. near \( x = c \).

To find the eqn. of a line, we need:
1. Slope
2. Point
$x_1, y_1; (c, f(c)) (c + ax, f(c + ax))$

$\frac{y_2 - y_1}{x_2 - x_1}$

Slope between $x_1$ and $x_2$

$\Delta x = \text{Change in } x$

Secant lines

To find slope: Approx. using slopes of

Steps:

1. $y = f(x)$
2. Point: $(c, f(c))$
3. $x = c$
4. Curve $y = f(x)$

Given equation:

$b) \text{Point-Slope: } y - y_1 = m(x - x_1)$

$8) \text{Use a slope-to-intercept: } y = mx + b$
\[ m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{f(c + \Delta x) - f(c)}{(c + \Delta x) - c} \]

\[
m = \frac{f(c + \Delta x) - f(c)}{\Delta x}
\]

Let \( \Delta x \) get small.

The line increase.

\[
m = \frac{f(c + \Delta x) - f(c)}{(c + \Delta x) - c} = \frac{f(c + \Delta x) - f(c)}{\Delta x} \frac{\text{Change in } y}{\text{Change in } x}
\]

or

\[
= \frac{\Delta y}{\Delta x}
\]

\( \Delta y = \) change in \( y \).
Define if $f(x)$ is defined on an open interval containing $x = c$, if it exists.

Then the limit \( \lim_{{\Delta x \to 0}} \frac{f(x + \Delta x) - f(x)}{\Delta x} \) has slope $m = \frac{f(x)}{x^2}$ at point $x = c$. 

Is the slope of the line to the graph of $f$ at point $x = 2$. 

Evaluate $f(2) = x^2 + 4x + 3$ 

\[ f(2 + \Delta x) = (2 + \Delta x)^2 \] 

Ex: Find the slope of $f(x) = x^2$ at point $x = 2$. 

$\tan m = \lim_{{\Delta x \to 0}} \frac{f(2 + \Delta x) - f(2)}{\Delta x}$
\[
\lim_{\Delta x \to 0} \frac{4 + 2\Delta x + 2\Delta x + 6}{(2 + \Delta x)^2} = \lim_{\Delta x \to 0} \frac{4\Delta x + 2\Delta x}{(2 + \Delta x)^2} = 4 + 0 = 4
\]