Problem 1: Find the **instantaneous rate of change** at \( x = 3 \) for \( f(x) \)

\[
\text{Find } f'(x) \\
\text{Find } f'(3) \quad \text{(Plug in } x = 3) 
\]

Problem 2: Find the **average rate of change** between \( x = 2 \) and \( x = 3 \) for \( f(x) \).

\[
\text{Slope between } (2, f(2)) \text{ and } (3, f(3)) \\
\frac{f(3) - f(2)}{3 - 2} = \frac{f(b) - f(a)}{b - a}
\]
Chain Rule (Thm 2.10)
If \( f(u) \) is a diff. fn. of \( u \) and \( u = g(x) \)
is a diff. fn. of \( x \) \[ f(g(x)) \] then
\( y = f(g(x)) \) is a diff. fn. of \( x \) and
\[
\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx}
\]
or
\[
\frac{d}{dx} \left[ f(g(x)) \right] = f'(g(x)) \cdot g'(x)
\]

Proof: Let \( h(x) = f(g(x)) \).
Show \( h'(c) = f'(g(c)) \cdot g'(c) \)
\[
h'(c) = \lim_{x \to c} \frac{h(x) - h(c)}{x - c} \quad \text{sub in } h(x) = f(g(x))
\]
\[ \lim_{x \to c} \frac{f(g(x)) - f(g(c))}{g(x) - g(c)} \]

\[ = \lim_{x \to c} \frac{f'(g(x)) \cdot g'(x)}{g(x) - g(c)} \]

\[ = \lim_{x \to c} \frac{f'(g(x)) \cdot g'(x)}{g(x) - g(c)} \]

Find the derivative at 3:
\[ g'(x) = \frac{(1+x)^2}{(1-x)^2} \]

\[ g'(3) = \frac{(1+3)^2}{(1-3)^2} = \frac{4^2}{(-2)^2} = \frac{16}{4} = 4 \]
\[ \begin{align*}
= & \ 3 \left( \frac{1+x}{1-x} \right)^2 \left( \frac{1-x + 1+ x}{(1-x)^2} \right) \\
= & \ 3 \cdot \frac{(1+x)^2}{(1-x)^4} \cdot \frac{2}{(1-x)^2} \\
= & \ 6 \frac{(1+x)^2}{(1-x)^2} \cdot \frac{1}{(1-x)^2} = 6 \frac{(1+x)^2}{(1-x)^4}
\end{align*} \]

Sec. 2.5

Implicit Differentiation.

So far (derivatives) Given \( y = f(x) \)

Use rules to find \( \frac{dy}{dx} = f''(x) \)

What if we want the slope of tan line at \( x = 0 \) to \( x^2 + y^2 = 9 \)?

\[ \begin{align*}
\ y^2 &= 9 - x^2 \\
\ y &= \pm \sqrt{9-x^2}
\end{align*} \]
$x^2 y + y^2 = -5x$

Can we solve for $y$?

Probably not.

What about $x^2 y + y^2 = -5x$?

If you can write $y = f(x)$, explicit form.

Implicit form (not in $y = f(x)$).

$x^2 y + y^2 = -5x$

$\frac{d}{dx} \left[ y^2 \right] = 2y \cdot \frac{dy}{dx}$

$x^2 = \int \left[ y^2 \right] dx$

Goal: Start implicit form and find $\frac{dy}{dx}$.

We will need to use this idea: implicit.

Assume $y = f(x)$.

$\frac{d}{dx} \left[ f(x)^2 \right] = 2\cdot f(x) \cdot f'(x)$
\[ \frac{d}{dx} \left[ \cos(y) \right] = -\sin(y) \cdot \frac{dy}{dx} \]

Steps:
1. Implicit diff.
2. Finding \( \frac{dy}{dx} \). Assume \( y = f(x) \).
3. Diff. terms/factors with \( x \)'s as usual.
4. Diff. terms/factors with \( y \)'s using Chain Rule. (multiply by \( \frac{dy}{dx} \)).
5. Solve for \( \frac{dy}{dx} \).

ex: Find \( \frac{dy}{dx} \)
1. \( x^2 - y^2 = 2 \)
\[
\frac{d}{dx} \left[ x^2 - y^2 \right] = \frac{d}{dx} [2]
\]
\[
2x - 2y \frac{dy}{dx} = 0 \quad \text{Chain rule on y term}
\]
\[
+ 2y \frac{dy}{dx} + 2y \frac{dy}{dx}
\]
\[
\text{Solve for } \frac{dy}{dx}
\]

\[
\frac{dx}{2y} = \frac{2y}{2y} \frac{dy}{dx}
\]
\[
\frac{dy}{dx} = \frac{x}{y}
\]

2) \[
\sec(y) = x - y
\]
\[
\frac{d}{dx} \left[ \sec(y) \right] = \frac{d}{dx} \left[ x - y \right]
\]
\[
\sec y \tan y \cdot \frac{dy}{dx} = 1 - \frac{dy}{dx}
\]
\[
\text{Solve for } \frac{dy}{dx}
\]
\[ \frac{dy}{dx} + \tan y \frac{dy}{dx} = 1 \]

\[ \frac{dy}{dx} \left( \sec y \tan y + 1 \right) = 1 \]

\[ \frac{dy}{dx} = \frac{1}{\sec y \tan y + 1} \]
\[ s(t) = -16t^2 + 100t + 240 \]

Use a table to estimate

\[ \lim_{t \to 2} \frac{s(t) - s(2)}{t - 2} \]

Let \( y_1 = -16x^2 + 100x + 240 \)

In \( y_2 = \frac{s(t) - s(2)}{t - 2} = \frac{(y_1(x) - y_1(2))}{(x-2)} \)

Table as \( x \to 2 \)

Due next Wed.