Exam 2 returned on Friday

Sec. 3.1 Covered Friday (2-20)

Quick Review of Sec. 3.1

1. Minimum: We say \( f(c) \) is a minimum value when \( f(c) \leq f(x) \) for \( x \in I \) (interval I)

2. Maximum (max.): \( f(c) \) is a max. value when \( f(c) \geq f(x) \) for \( x \in I \)

max/min known as extrema values (aka "extrema")

(Global)

Absolute max/min ex: \( y = x^2 - 4 \)

Absol. min at \((0, -4)\)

no absol. max
ex: \( y = f(x) = x^3 \)

no absd. max or min.

note cl: \((-\infty, \infty)\) open interval.

Extreme Value Thm.

If \( f(x) \) is cont. on a closed 
then \( f \) has both a 
max & min 
on \([a,b]\).

\( I = \text{interval} = [a,b] \)

Steps to find a max/min on a closed interval

1. Find the critical \#s of \( f(x) \) on \((a,b)\).

   \text{When } f'(x) = 0 \text{ or DNE}

   look at \#s

   in \((a,b)\).

2. Create a list: Plug critical \#s into \( f(x) \)

3. Plug \( x = a, x = b \) into \( f(x) \): \( f(a) \)

4. Smallest numbers \((2 \& 3)\) \( \text{min} \)

5. Largest number \((2 + 3)\) \( \text{max} \)
ex: \( f(x) = 2 \sin(x) - \cos(2x) \) on \([0, 2\pi]\) ③

Find the max/min (or find extrema)

1. \( f'(x) = 2 \cos(x) - (-\sin(2x) \cdot 2) \)
2. \( f''(x) = 2 \cos(x) + 2 \sin(2x) = 0 \)
   
   Inputs not equal
3. \( \sin(2x) = 2 \sin(x) \cos(x) \)

2 \cos(x) + 2 \left( 2 \sin(x) \cos(x) \right) = 0

2 \cos(x) + 4 \sin(x) \cos(x) = 0

2 \cos(x) \cdot (1 + 2 \sin(x)) = 0

2 \cos(x) = 0 \quad \text{or} \quad 1 + 2 \sin(x) = 0 \quad \text{[0, 2\pi]}

\begin{align*}
\frac{8 \sin(x) = -1}{2} & \quad 3\text{rd} \text{ & } 4\text{th} \text{Q.} \quad \text{Solve} \\
& \quad x = \frac{7\pi}{6} \quad \frac{11\pi}{6}
\end{align*}
(2) \( f\left(\frac{3\pi}{2}\right) = -3 \) \( \text{max} \)
\( f\left(\frac{7\pi}{6}\right) = -1 \)
\( f\left(\frac{11\pi}{6}\right) = -1.5 \) \( \text{min} \)

(3) \( f(0) = -1 \)
\( f(2\pi) = -1 \)

Sec. 3.2
Rolle's Theorem and Mean Value Theorem (MVT)

"Mean" \( \leftrightarrow \) "Average" Value

Rolle's Theorem:
Let \( f(x) \) be a cont. fn. on \([a, b]\) and diff. on \((a, b)\). If \( f(a) = f(b) \) then there is at least one \( c \in (a, b) \) such that \( f'(c) = 0 \).
Graph of Rolle's Thm

\[ f'(c) = 0 \] (horizontal tan line)

Proof see Book.

ex: \( f(x) = x^2 + 6x \).

Find the 2 x-intercepts and find \( c \) between the 2 x-inter. Where \( f'(c) = 0 \) (guaranteed by Rolle's Thm).

1. \( f(x) = x^2 + 6x \) is cont. (poly.)
2. \( f(x) \) is diff. (poly).
3. Show \( f(a) = f(b) \) \( a, b \) are x-inter.

\[ 0 = x^2 + 6x \]
\[ 0 = x(x + 6) \]
\[ x = 0, x = -6 \]
\[ f(-e) = -0 \]
\[ f(x) = 0 \]
\[ f(a) = 0 \]

Rolle's Theorem can be used.

\[ f'(x) = 2x + 4 \]
\[ x = -3 \]

\[ c = -3 \]

\[ \int_{-2}^{4} (x^2 + 4x) \, dx = 0 \]

\[ f(x) = (x-4)(x+4)^2 \]
\[ g(x) = \frac{1}{2}x \]

Find \( c \) such that Rolle's Theorem applies and find

\[ f(x) = (x-4)(x+4)^2 \]
\[ f'(x) = (x-4)(x+4) \]
\[ f(c) = 0 \]

1. \( f(x) \) is continuous on \([a, b]\).
2. \( f(x) \) is differentiable on \((a, b)\).
3. \( f(a) = f(b) = 0 \).

Show Rolle's Theorem applies and find \( c \).
We want the

$\frac{e}{2}$ or $x = 2$

$(-3, 4)$

$C = 2$

$C \in (a, b)$ such that

$\text{MVT}$

Mean Value Theorem

If $f(x)$ is cont. on $[a, b]$ then
$f'(c) = \frac{f(b) - f(a)}{b - a}$

$f(x) = (x+2)^2 - (x+2)(x-4) + \frac{(3x-6)(x+2)}{2}$

$x^2 + 4x + 4 - x^2 + 2x - 8 + x + 2$

$x^2 + 6x - 2 = 0$

$x = -2, 1$

$\text{Soln.}$

$\text{Det. element}$
\[
\frac{f(b) - f(a)}{b - a} = \text{slope of secant line}
\]

At some pt. \( x = c \)

By definition of derivative, \( \frac{f(b) - f(a)}{b - a} = \frac{y_2 - y_1}{x_2 - x_1} \)

Proof:

Find eqn. of line:

\[
m = \frac{y_2 - y_1}{x_2 - x_1}
\]

\[
y - y_1 = m(x - x_1)
\]

\[
y = f(x) = f(a)
\]

Hence:

\[
\frac{y - f(a)}{x - a} = \frac{f(b) - f(a)}{b - a}
\]
(a) \[
\left[ \frac{(f(b) - f(a))}{b-a} \right] (x-a) + f(a) \\
= \left[ \frac{f(b) - f(a)}{b-a} \right](x-a) + f(a)
\]

Secant line through endpoints.

Create a new fn.

Apply Rolle's Thm to \( g(x) \) if possible.

1. \( g(a) = f(a) \)
2. \( g(b) = f(b) \)
3. \( g(a) = f(a) \) is cont.
4. \( g(x) \) is diff.
5. \( g(x) \) is const.

\[
elimit_{x \to a} \left[ \frac{f(b) - f(a)}{b-a} \right] = 0
\]
$y = \text{line}$

$\frac{dy}{dx} = \text{slope}$

$g(c) = 0$ is a line.

$g(x) = f'(x)$

$g(x) = \frac{f(x) - f(a)}{b-a}$

$g(x) = \frac{f(x) - f(a)}{b-a}$

$0 = g(c) = f'(c) = \frac{f(b) - f(a)}{b-a}$

Your goal is to find $c$ in $(a, b)$.

Note: $f(x) = x^4 - 8x + f(27)$. Show that $c$ applies and $c$ is guaranteed by MVT.

**Example**:

$f(x) = x^4 - 8x + f(27)$
\( f(x) \) is cont. / diff. blc poly.

Find \( c \in (0, 2) \) s.t. \( f'(c) = \frac{f(b) - f(a)}{b - a} \).

\[
f'(x) = 4x^3 - 8
\]

Set \( f'(x) = \text{slope} \)
\[
4x^3 - 8 = 0
\]
\[
4x^3 = 8
\]
\[
x^3 = 2
\]
\[
x = \sqrt[3]{2} \approx 1.26 \in (0, 2)?
\]

Sec. 3.1 + 3.2 Due Sunday @ 11:59 pm.

All odd problems are worked out on

www.calcchat.com

Tip: right-click - open in new tab for full size.