Sec. 3.8

Newton's method

Finding roots \(\equiv\) zeros \(\equiv\) x-intercepts for a fn. can be very difficult.

Polynomials

\[\text{deg} = 1 \quad \text{(linear)} \quad mx + b = 0\]

\[\text{deg} = 2 \quad \text{(quad. formula)} \quad ax^2 + bx + c = 0\]

\[x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}\]

\[\text{deg} = 3 \quad \text{(cubic poly)} \quad \text{There is a cubic formula.}\]

\[\text{What we do: Rational Roots Thm}\]

\[\text{List of candidates}\]

\[\text{Plug into f(x)}\]

\[\text{If f(x) = 0 \implies root.}\]

\[\text{deg} = 4 \quad \text{There is a quartic formula.}\]
Galois proved for a poly. of deg \( \geq 5 \) there are no closed formulas to find roots.

**Newton's Method** is used to find (approximate) value of roots. It finds roots quickly.

**Idea** cont. \( f(x) \) on \( [a,b] \) and diff. \( f(x) \) on \( (a,b) \).

\[
\begin{align*}
\text{root at } x &= c \\
\text{tan line to } f(x) \text{ at } x_i
\end{align*}
\]
Algorithm: \( f_n(x) \) derivative \( f(x) \)

1. Step 1: Initial guess \( x = x_0 \)
2. Step 2: Eqn. of tan line \( (x_1, f(x_1)) \) \( m = f'(x_1) \)
3. Step 3: \( y - f(x_1) = f'(x_1)(x - x_1) \)
   \[ 0 - f(x_1) = f'(x_1)(x - x_1) \]
   \[ x = x_1 + \frac{f(x_1)}{f'(x_1)} \]

Repeat steps 1, 2, 3.
In General:

\[ x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \quad \text{for when} \quad f'(x_n) = 0 \]

Sec.3.9

1. Monday after Spring Break - do
   In 15 min - then do online hw
   or
2. I scan in lecture notes - post online
   post on website
   email to class.
(3) Sec. 3.7
distance between
f(x) = pt. net
on fix.

Sec. 3.1
note

open intervals
1st Derivative
t ≠ 2

1st Derivative
t ≠ 2

2nd Derivative
t ≠ 2

anst 1
-2x² + 2 = 0
x = ± 1
x = ± 1

anst 1

[-2, 2]
To find min, take derivative test:

\[ D(x) = \frac{d}{dx} \left[ (x-2)^2 + (x^2 - \frac{1}{2})^2 \right] \]

Also check to min \( P \sim (2, \frac{1}{2}) \)

\[ |D| = \sqrt{1 \cdot (x^2 - x)^2 + (x^2 - y)^2} \]

Graph shows that is closest to \( P \). The goal is minimize the distance of \( f(x) = x \) from \( P \). Find the point on the graph.
just like #25 in Sec. 3.7

What length & width should the rectangle have so the area is a max?

max \( A = 2xy \)

2 variables \( y = \sqrt{25-x^2} \)

\( A(x) = 2x \sqrt{25-x^2} \)

\( 0 \leq x \leq 5 \)

Sec. 3.1
Find eqn. of slant asymptote

\[ f(x) = \frac{x^2 - 2x + 3}{x + 1} \]

\[ \text{Deg (num)} = 1 + \text{deg (num)} = 1 + \text{deg (denom)} = 1 \]

\[ x + 1 \]

\[ x^2 - 3x + 3 \]

\[ x + 1 \]

\[ x + 1 \]

\[ x^2 - 2x + 3 \]

\[ \frac{x^2 - 2x + 3}{x + 1} \]

\[ \frac{x - 3}{x + 1} \]

\[ \frac{-3x + 3}{x + 1} \]

\[ \frac{-3x + 3}{x + 1} \]

\[ \frac{6}{x + 1} \]

\[ \frac{(x - 3)}{x + 1} \]

\[ \frac{6}{x + 1} \]

\[ \frac{6}{x + 1} \]
\[ f(x) = 7 - 6x - x^3 \]

- **x-inter**
  - \( x = 1 \) \( \begin{align*} y &= f(1) = 7 - 6(1) - (1)^3 = 0 \\
&= 1 \end{align*} \)
  - \( x = -1 \) \( \begin{align*} y &= f(-1) = 7 - 6(-1) - (-1)^3 = 0 \\
&= 0 \end{align*} \)
  - \( x = 0 \) \( \begin{align*} y &= f(0) = 7 - 6(0) - (0)^3 = 0 \\
&= 0 \end{align*} \)

- **y-inter**
  - \( y = 7 \) \( \begin{align*} x &= 0 \end{align*} \)
  - \( y = 0 \) \( \begin{align*} x &= \pm 2 \end{align*} \)

- **D:** \( (-\infty, \infty) \)

- **E:**
  - \( x = 0 \) \( \begin{align*} y &= 0 \end{align*} \)
  - \( x = -1 \) \( \begin{align*} y &= 0 \end{align*} \)
  - \( x = 1 \) \( \begin{align*} y &= 0 \end{align*} \)

- **G:**
  - \( x = 0 \) \( \begin{align*} y &= 7 \end{align*} \)
  - \( x = -1 \) \( \begin{align*} y &= 0 \end{align*} \)
  - \( x = 1 \) \( \begin{align*} y &= 0 \end{align*} \)

- **H:**
  - \( x = 0 \) \( \begin{align*} y &= 7 \end{align*} \)
  - \( x = -1 \) \( \begin{align*} y &= 0 \end{align*} \)
  - \( x = 1 \) \( \begin{align*} y &= 0 \end{align*} \)

- **K:**
  - \( x = 0 \)
  - \( x = -1 \)
  - \( x = 1 \) \( \begin{align*} y &= 7 \\
&= 1 \\
&= 0 \end{align*} \)
No asymptotes. We plop.

\[ f(x) = 7 - 6(-x) - 3(-x)^3 = 7 + 6x + x^3 \]

No extrema. No extrema.

f"(x) = -6g(x) = 6

\[ f''(1) = -6g(1) = 6 \quad 1 \]

\[ f''(x) = -6g(x) = 6 \quad x = 0 \]

Inflection pt. x = 0.