The Kaprekar Number

1. Select any four digit number with unique (different) digits.

   \[ 6172 \]

2. Rearrange the digits so that the largest and smallest possible numbers are formed.

   Largest = \[
   \begin{array}{c}
   7621 \\
   6543 \\
   4530 \\
   \end{array}
   \]

   Smallest = \[
   \begin{array}{c}
   1267 \\
   2358 \\
   3456 \\
   3758 \\
   \end{array}
   \]

3. Subtract the smallest number from the largest number to obtain another four digit number.

   \[
   8352 - 2358 = 60174
   \]

4. Repeat steps 1 through 3 with the new number. Repeat until you get the Kaprekar number of 6174. This will take 7 or fewer steps. How many steps did it take you?
Solving Optimization Problems

A rancher has 200 ft. of fencing to enclose an area. What dimensions should be used so that the enclosed area is a maximum?

\[3y + 4x = 200 \quad \text{maximize area} \]

\[\text{Area} = xy \quad \text{subject to} \]

In this case, we want to maximize the area. We need to use one variable to get down to 2 variables.
\[ \frac{\partial^2}{\partial y^2} \left( 200y - 3y^2 \right) = y - \frac{3y^2}{2} \]

For max: \[ A(x, y) = 2 \left( \frac{200 - 3y}{y^2} \right) \]

\[ A_y(y) = 100 - \frac{3y}{2} \]

\[ A''(y) = 100 - \frac{3y}{2} \]

Use 1st or 2nd derivative test to prove this is a max.

\[ A''(y) = 100 - \frac{3y}{2} \]

\[ A''(y) = -3 < 0 \]

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Key: $\frac{3}{5}$

\[ y = \frac{20 - 3x}{4} \]

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\[ x = 25 \text{ ft.} \]

\[ \frac{200 - 3x}{4} \]

Dimensions: \[ x = \frac{200 - 3x}{4} \]

Equilateral triangles equal and a square

\[ A = \frac{\sqrt{3}}{4} x^2 \]

1. Set up perimeter eqn. \[ 20 = 3x + 4y \]

2. Set up area area

How much wire should be used to make a square

Shape so that the total enclosed area is a maximum?
\[ A(x) = \frac{\sqrt{3}}{4} x^2 + \left( \frac{20-3x}{4} \right)^2 \]

**Chain Rule**

\[ A'(x) = \frac{\sqrt{3}}{4} (2x) + 2 \left( \frac{20-3x}{4} \right) \cdot \left( -\frac{3}{4} \right) \]

**Derivative is slope**

\[ A'(x) = \frac{\sqrt{3}}{2} x - \frac{3}{8} \left( 20 - 3x \right) = 0 \]

\[ 8 \cdot \left( \frac{\sqrt{3}}{2} x - \frac{15}{2} + \frac{9}{8} x \right) = 0 \cdot 8 \]

\[ 4\sqrt{3} x - 60 + 9x = 0 \]

\[ x(4\sqrt{3} + 9) = 60 \]

\[ x = \frac{60}{4\sqrt{3} + 9} \]

\[ A''(x) = \left( \frac{\sqrt{3}}{2} + \frac{9}{8} \right) \text{ POS.} \quad \text{hm...} \]
\[ A \left( \frac{3a}{2b} \right) = \frac{a}{2} \]

\[ A \left( \frac{c}{2a} \right) = \frac{c}{2} \]

\[ A \left( 4 \right) = 2 \]

\[ \text{max} \]

\[ x = 0 \]

\[ x = \frac{3}{2c} \]

All line for square

All line for \( A \)

20 m of \( h \)

So the original problem is on a closed interval.
Sec. 3.6

Graph \( f(x) = \frac{1}{x-3} + 2 \)

I.) Domain, intercepts, symmetry, horizontal asymptotes, vertical asymptotes

Hint \( f(x) = \frac{1}{x-3} + \frac{2(x-3)}{(x-3)} = \frac{2x-5}{x-3} \)

II.) From \( f'(x) \) find open intervals of inc./dec.
Identify all rel. extrema. Write as points \((x, y)\)

III.) From \( f''(x) \) find open intervals where concave up/down. Identify all inflection pts. \((x, y)\).

IV.) Graph.
Sln.

I) Domain \( \{ x \mid x \in \mathbb{R}, x \neq 3 \} \)

**x-intercept** \((y=0)\) \(2x-5=0\)
\(x=5/2\) \((5/2,0)\)

**y-intercept** \((x=0)\) \(y=f(0)=5/3\)
\((0,5/3)\)

**Sym?**
\(f(-x) = \frac{2(-x)-5}{(-x)-3} = \frac{-2x-5}{-x-3} = \frac{2x+5}{x+3} = \pm f(x)\) no sym. about y-axis

**H.A.**
\(\lim_{x \to \pm\infty} \frac{2x-5}{x-3} = \lim_{x \to \pm\infty} \frac{\frac{2x}{x} - \frac{5}{x}}{1 - \frac{3}{x}} = \frac{2}{1} = 2\)

**V.A.**
\(x=3\)

H.A. is the line \(y=2\)

II) \(f'(x) = \frac{(x-3)(2) - (2x-5)(1)}{(x-3)^2} = \frac{2x-6-2x+5}{(x-3)^2} = \frac{-1}{(x-3)^2} = \frac{-1}{3}\)

\(f'(0) = \frac{-1}{9} < 0\)
\(f'(4) = \frac{-1}{(1)^2} = -1 > 0\)

dec. \((-\infty,3)\) \((3,\infty)\) no rel. max/min blc \(x=3\) is not in the domain of \(f(x)\) (original fn.)

III) \(f'(x) = (-1)(x-3)^{-2}\)
\(f''(x) = (-1)(-2)(x-3)^{-3} = \frac{2}{(x-3)^3}\)

\(f''(0) = \frac{2}{-2} < 0\)
\(f''(4) = \frac{2}{(4-3)^3} = 2 > 0\)

no inflection pt. blc \(x=3\) is not in the domain of \(f(x)\)