Web Assign

- Update Flash
- Google Chrome
  or Firefox
- don't use Internet Explorer (IE)
- WebAssign Tech Help

  Help

  - Phone number – answer until 5 pm PST
  - Email them.

Deadline for Sec. 1.3 – HW due next Thursday.

Query 1

  next Wed.
  2 Questions
  Sec. 1.1 and 1.2.
Sec. 1.1

1. a) Calculus
   - v(t) = 20 + 4 \cos (t)
   - velocity is not constant
   - t = 30 sec
   - (40, 45°)
   - t = 30 sec
   - \( r = v(t) \)

   b) Estimate the distance traveled numerically (table)
   - \( d = r \cdot t \)

<table>
<thead>
<tr>
<th>t</th>
<th>v(t)</th>
<th>y(t)</th>
</tr>
</thead>
<tbody>
<tr>
<td>29.99</td>
<td></td>
<td></td>
</tr>
<tr>
<td>30.1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>30.01</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\( y_1 = \frac{20 + 4 \cos (t)}{t} \cdot y(t) \)
Section 1.2 (Cont.)

When limits DNE

1. At a vertical asymptote — fn. values $\to +\infty$ or $-\infty$

2. When the 2 one-sided limits are not equal.

3. When the fn. oscillates.

ex: $\lim_{x \to 0} \sin \left( \frac{1}{x} \right)$

→ Use graph or table
→ Look at values near 0
→ Outputs (fn. values) never settle down

⇒ Limit DNE.
Formal Definition of a Limit

We want to make precise:

$$\lim_{x \to c} f(x) = L$$

So far, "As x gets closer and closer to c, f(x) gets closer and closer to L".

Example: let $f(x) = x + 2$

What is $\lim_{x \to 2} f(x)$?

$$\lim_{x \to 2} f(x) = 4.$$ 

Question: How close to 2 does x have to be so $f(x)$ differs from 4 by less than 0.01?
(y = x + 2)

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a) When y = 4.01, what is x?

b) When y = 3.99, what is x?

x has to be within 0.01 of 2 (either x = 1.99 or x = 2.01)

1.5 - 1 = 1 - x

Recall Distance from a to b is |a - b|
\[ \text{Distance from } x \text{ to } 2 = |x-2| \]

\[ \text{Distance from } f(x) \text{ to } 4 = |f(x)-4| \]

Q: How close does \( x \) have to be to 2?

A: \( x \) has to be within 0.01 of 2

\[ |x-2| < 0.01 \]

Q: Translate using absolute values:

\( f(x) \) is within 0.01 of 4

\[ |f(x)-4| < 0.01 \]
Absolute Value Facts

1. Distance between \( a \) and \( b \) is
\[ |b - a| = |a - b| \]

2. \(|a \cdot b| = |a||b|\)

3. \(|\frac{a}{b}| = \frac{|a|}{|b|}\)

4. (Triangle Inequality)
\[ |a + b| \leq |a| + |b| \]

5. \(|w| < a \Rightarrow -a < w < a\)

6. \(|w| > a \Rightarrow w > a \text{ or } w < -a\)

Examples:
\[ 12x + 1 < 3 \Rightarrow -3 < 2x + 1 < 3 \]
Formal Definition of a Limit

Let \( f(x) \) be a function defined on an open interval \( (a, b) \). Let \( L \) be a real number. Then, for each \( \varepsilon > 0 \), there exists a \( \delta > 0 \) such that for each \( x \) with \( 0 < \left| x - c \right| < \delta \), we have

\[
\left| f(x) - L \right| < \varepsilon
\]

Identify how close does \( x \) have to be to \( c \) and \( f(x) \) is within \( 0.01 \) of \( L \).
If \( |f(x) - L| < \varepsilon \)
\[ f(x) - 1 < 0.01 \] 

\( \varepsilon = 0.01 \) 

"error"

**New answer**

"How close does \( x \) have to be to 2..."

\[ y = 2x + 4 \]
\[ y = 8.0 \]
\[ x = ? \]
\[ x = 2.005 \]

\[ y = 7.99 \]
\[ x = ? \]
\[ x = 1.995 \]

\[ |x - 2| < 0.005 \]
\[ \delta = 0.005 \]
\[ |x - c| < \delta \]
Given \( y = f(x) \)

- \( \lim_{x \to c} f(x) = L \)
- \( \varepsilon \)

**Your job** Find \( \delta \)

**Ex:**\( \lim_{x \to 1} (-2x + 4) = 2 \)

Find \( \delta \) so that \( |f(x) - L| < 0.001 \)

\( |x - c| < \delta \)

\( y = 2.001 \quad x = 0.9995 \)

\( y = 1.999 \quad x = 1.0005 \)

\( \delta = 0.0005 \)
Ex: Use the \( \varepsilon - \delta \) definition of limit to prove \( \lim_{x \to c} f(x) = L \):
\[
\text{Let } L = 5, \quad f(x) = \frac{x+2}{x+1}, \quad c = 3
\]

Find \( \delta \): When \( |x - c| < \delta \)
\[
|x - c| < \delta 
\]

Assume \( \varepsilon > 0 \)
\[
|f(x) - L| < \varepsilon
\]
\[
|f(x) - 5| < \varepsilon
\]

Simple: Simplify
\[
\frac{3}{x+2} - 5| < \varepsilon
\]

Look for factors of \( |x-3| \) to find \( \delta \).
\[ |x - 3| < 8 \]

Put the goal back to \( x \).

Let \( s = 8 \).

\[ 3 - 8 < x < 3 + 8 \]

\[ -5 < x < 11 \]