Problem 1: (20 points) Let \( f(x) = x^2 + 1 \). Use the limit process to find the area between the graph of \( f(x) \) and the x-axis on the interval \([0,3]\). You must show all of your work. Include a sketch of the region. You must tell me what \( \Delta x \) is, as well as the value you used for \( c \).

\[
\Delta x = \frac{3-0}{n} = \frac{3}{n}
\]

Right endpoint: \( c = \frac{3}{n} \)

\[
f(c) \Delta x
\]

\[
\lim_{n \to \infty} \sum_{i=1}^{n} \left[ \left( \frac{3i}{n} \right)^2 + 1 \right] \left( \frac{3}{n} \right)
\]

\[
= \lim_{n \to \infty} \sum_{i=1}^{n} \left( \frac{27i^2}{n^3} + \frac{3}{n} \right)
\]

\[
= \lim_{n \to \infty} \left( \frac{27}{n^3} \frac{n(n+1)(2n+1)}{6} \right) + \left( \frac{3}{n} \right)
\]

\[
= \frac{54}{6} + 3 = 9 + 3 = 12
\]
Problem 2: (30 points) Evaluate the integral. Pick any 3 out of the 4 to do. Circle the three you want me to grade. If you do not circle the ones you want graded, I will grade parts a, b, and c.

a) \[ \int (-3x^5 + 2x - \frac{1}{x}) \, dx = \int (-3x^5 + 2x - x^{-\frac{1}{2}}) \, dx \]
\[ = -\frac{3x^6}{6} + \frac{2x^2}{2} - \left(2x^{\frac{1}{2}}\right) + C \]
\[ = -\frac{1}{2}x^6 + x^2 - 2x^{\frac{1}{2}} + C \]

b) \[ \int \frac{\sec^2 x}{\tan x} \, dx \quad \text{he} + u = \tan x \quad du = \sec^2 x \, dx \]
\[ = \int \frac{1}{u^3} \, du = \int u^{-3} \, du = \frac{u^{-2}}{-2} + C = -\frac{1}{2\tan^2 x} + C \]

c) \[ \int x^3(x^4 - 16)^{19} \, dx \]
\[ = \int x^3 u^{19} \left( \frac{du}{4x^3} \right) \]
\[ = \frac{1}{4} u^{19} + C = \frac{1}{76} (x^4 - 16)^{19} + C \]

d) \[ \int \frac{\cos(\sqrt{x})}{\sqrt{x}} \, dx \]
\[ = \int \frac{\cos(u)}{\sqrt{u}} \left( 2\sqrt{u} \, du \right) = 2 \frac{\sin(u)}{u} + C \]
\[ = 2 \sin(1/x^\frac{1}{2}) + C \]
Problem 3: (10 points) You are given that \( \int_{0}^{9} f(x) \, dx = 5 \) and \( \int_{3}^{9} f(x) \, dx = -1 \). Find

a) \( \int_{0}^{3} f(x) \, dx = \int_{0}^{3} f(x) \, dx + \int_{3}^{9} f(x) \, dx \)
\[\text{Ans} = 6\]

b) \( \int_{3}^{9} f(x) \, dx = - \int_{3}^{9} f(x) \, dx = - (-1) = 1\]

c) \( \int_{0}^{5} f(x) \, dx = 5 \int_{0}^{9} f(x) \, dx = 5 \cdot 5 = 25\)

Problem 4: (10 points) Pick either part a or part b to do. You only need to do one.

a) Let \( F(x) = \int_{2}^{x} \frac{1}{1+t^4} \, dt \). Find \( F'(x) \) and \( F'(3) \).
\[ F'(x) = \frac{1}{1+x^4} \]
\[ F'(3) = \frac{1}{1+3^4} = \frac{1}{82} \approx 0.012195122 \]

b) Find the average value of \( f(x) = 2x^2 + 3 \) on the interval \([2,4]\).
\[ = \frac{1}{2} \int_{2}^{4} (2x^2 + 3) \, dx = \frac{1}{2} \left[ \frac{2}{3} x^3 + 3x \right]_{2}^{4} \]
\[ = \frac{1}{2} \left[ \left( \frac{2}{3} \cdot 64 + 12 \right) - \left( \frac{2}{3} \cdot 8 + 6 \right) \right] \]
\[ = \frac{1}{2} \left( \frac{130}{3} \right) = \frac{65}{3} \approx 21.67 \]
Problem 5: (30 points) Evaluate the integral. Pick any 3 out of the 4 to do. Circle the three you want me to grade. If you do not circle the ones you want graded, I will grade parts a, b, and c.

a) \( \int_0^\pi (\cos(x) + 1) \, dx = \sin x + x \Bigg|_0^\pi = (\sin \pi + \pi) - (\sin 0 + 0) = \pi \)

b) \( \int_0^1 x \sqrt{x^2 + 1} \, dx \)

\[
\text{let } u = x^2 + 1 \quad \frac{du}{dx} = 2x \implies du = 2x \, dx
\]

\[
x = 0 \quad u = 0^2 + 1 = 1
\]

\[
x = 1 \quad u = 1^2 + 1 = 2
\]

\[
= \int_1^{3/2} u^{1/2} \left( \frac{du}{2x} \right) = \frac{1}{2} \left( \frac{u^{3/2}}{3} \right) \bigg|_1^{3/2} = \frac{1}{2} \left( \frac{2^{3/2}}{3} - 1 \right) \approx 0.609
\]

c) \( \int_2^5 2x^{-3} \, dx = \int_2^5 2x^{-3} \, dx 
\]

\[
= -\frac{1}{x^2} \bigg|_2^5 = -\frac{1}{25} - (-\frac{1}{4}) = \frac{1}{4} - \frac{1}{25} = \frac{21}{100}
\]

d) \( \int_0^4 (x - 2) \, dx \)

\[
= \int_0^2 (x - 2) \, dx + \int_2^4 (x - 2) \, dx
\]

\[
= -\frac{X^2}{2} + 2x \bigg|_0^2 + \left( \frac{x^2}{2} - 2x \right) \bigg|_2^4
\]

\[
= (\frac{4^2}{2} - 4) - (0) + \left[ \frac{4^2}{2} - 8 \right] - \left( \frac{2^2}{2} - 2 \right)
\]

\[
= \left[ (4 - 4) - (0) \right] + \left[ (8 - 8) - (2 - 2) \right]
\]

\[
= 4 + 0 = 4
\]