Problem 1: (30 points) Find the derivative of each function. Pick any 3 out of the 4 to do. Circle the three you want me to grade. If you do not circle the problems you want graded, I will grade parts a, b and c.

a) \( g(x) = \log_5(5x^3 - 6) \)

\[
g'(x) = \frac{1}{\ln 5} \cdot \frac{15x^2}{5x^3 - 6}
\]

b) \( f(x) = \ln\left(-\frac{6x}{x^2 + 8}\right) \)

\[
f'(x) = \frac{1}{(x^2 + 8)} \cdot \frac{6}{x} - \frac{1}{x^2 + 8} \cdot 6x = \frac{-x}{x^2 + 8}
\]

c) \( h(x) = 4e^{3x} + 3^{3x} \)

\[
h'(x) = 4e^{3x} \cdot 3 + (\ln 3) \cdot 3^{3x} \cdot 3 = 12e^{3x} + 3(\ln 3) \cdot 3^{3x}
\]

d) \( f(x) = e^{\tan x} \)

\[
f'(x) = \frac{e^{\tan x} \cdot \sec^2 x}{\sec x e^{\tan x}}
\]
Problem 2: (30 points) Evaluate the integral. Pick any 3 out of the 4 to do. Circle the three you want me to grade. If you do not circle the ones you want graded, I will grade parts a, b, and c.

a) \[ \int \frac{1 - \cos(x)}{x - \sin(x)} \, dx \]
   \[ = \int \frac{1}{u} \, du = \ln|u| + C = \ln|x - \sin(x)| + C \]

b) \[ \int \frac{x^3}{x^2} \, dx \]
   \[ = \frac{e^4}{x^2} \left( -\frac{3x^2}{2} \right) = -\frac{1}{2} e^4 + C = \left\{ \frac{3x}{2} \right\} + C \]

\[ \int \left( 2 - \frac{1}{x+1} \right) \, dx = \left\{ 2x \right\} - 1 \ln|x+1| + C \]

\[ \lim_{a \to 0} \int_a^1 \frac{1}{x-1} \, dx \]
   \[ \left\{ \frac{x}{u} \int \frac{1}{u} \, du = \ln|u| \right\} \left\{ \frac{e}{1} \right\} = \ln(e) - \ln(1) = 1 \]
Problem 3: (20 points) Let \( f(x) = \sqrt{4x + 1} \)

a) What are the domain and range for \( f(x) \)?

\[ \text{Domain} \quad 4x + 1 \geq 0 \quad \Rightarrow x \geq -\frac{1}{4} \]
\[ \text{Range} \quad y \geq 0 \]

b) Is \( f(x) \) a 1-1 function? Why or why not?

Yes - it passes the horizontal line test.

c) Find \( f^{-1}(x) \). State its domain and range.

\[ x = \sqrt{4y + 1} \quad \Rightarrow x^2 = 4y + 1 \]
\[ \Rightarrow 4y = x^2 - 1 \quad \Rightarrow y = \frac{1}{4} (x^2 - 1) \]
\[ f^{-1}(x) = \frac{1}{4} (x^2 - 1) \]

d) Analytically verify that \( f(x) \) and \( f^{-1}(x) \) are inverses.

\[ f(f^{-1}(x)) = f \left( \frac{1}{4} (x^2 - 1) \right) = \sqrt{4 \left( \frac{1}{4} (x^2 - 1) \right)} + 1 \]
\[ = \sqrt{x^2} = x \quad \checkmark \]

\[ f^{-1}(f(x)) = \frac{1}{4} \left[ (\sqrt{4x + 1})^2 - 1 \right] \]
\[ = \frac{1}{4} [4x + 1 - 1] \]
\[ = \frac{1}{4} [4x] \]
\[ = x \quad \checkmark \]
Problem 4: (10 points) Pick either part a or part b to do. You only need to do one. Circle the one you want graded.

a) Let \( y = x^{x+1} \). Find \( \frac{dy}{dx} \).

\[
\frac{dy}{dx} = (x+1) \left( \frac{x}{x} \right) + \ln(x)
\]

\[
= x^{x+1} \left[ \frac{x+1}{x} + \ln(x) \right]
\]

b) Find \( \frac{dy}{dx} \) if \( \ln(xy) = x + y \).

\[
\frac{1}{xy} \left[ x \frac{dy}{dx} + y \cdot 1 \right] = 1 + \frac{dy}{dx}
\]

\[
x \frac{dy}{dx} + y = xy \left[ 1 + \frac{dy}{dx} \right]
\]

\[
x \frac{dy}{dx} = xy + xy \frac{dy}{dx} - y
\]

\[
\frac{dy}{dx} = \frac{xy - y}{x - xy}
\]

Problem 5: (10 points) A certain type of bacteria increases continuously at a rate proportional to the number present. If there are 500 present initially, and 1000 present 2 hours later, answer each of the following questions. Show your work.

a) Write the differential equation that models this situation.

\[
\frac{dP}{dt} = k \cdot P
\]

b) Solve the differential equation to find the function \( P(t) \), which gives the bacterial population when the input is time (in hours).

\[
\int \frac{dp}{p} = k \int dt
\]

\[
\ln(p) = kt + c
\]

\[
P(t) = Ce^{kt}
\]

\[
P(t) = 500e^{kt}
\]

\[
t = 0 \Rightarrow P(0) = 500
\]

\[
t = 2 \Rightarrow P(2) = 1000
\]

\[
k = \frac{\ln(1000)}{2} = 3.464
\]

\[
P(t) = 500e^{3.464t}
\]

\[
P(5.83) = 500e^{3.464 \cdot 5.83}
\]

\[
= 3772
\]