There are 5 problems on this exam, worth a total of 100 points. You must show all of your work for credit. No TI-89's or other calculators capable of symbolic manipulations allowed!

Problem 1: (30 points) Analytically find the derivative. Simplify and show your work.

a) \( f(x) = 5 \sec(x) \tan(x) \)

\[
f'(x) = \left( \sec(x) \right)' (\sec(x)) + \left( \tan(x) \right)' (5 \sec(x) \tan(x)) \\
= 5 \sec^3(x) + 5 \sec(x) \tan^2(x) \\
= 5 \sec(x) \left[ \sec^2(x) + \tan^2(x) \right] \\
= 5 \sec(x) \left[ 2 \sec^2(x) - 1 \right] \quad \text{or} \quad 5 \left[ \cos^2(x) + 2 \sin^2(x) \right] \\
\]

b) \( y = \frac{1-x}{x+2} \)

\[
\frac{dy}{dx} = \frac{(x+2)(-1) - (1-x)(1)}{(x+2)^2} = -x - 2 - 1 + x \quad \frac{1}{(x+2)^2} \\
= -\frac{3}{(x+2)^2} \\
\]

c) \( g(x) = \sin^2(4\pi x + 1) \)

\[
g'(x) = 2 \sin(4\pi x + 1) \cdot \frac{d}{dx} \left[ \sin(4\pi x + 1) \right] \\
= 2 \sin(4\pi x + 1) \cos(4\pi x + 1) \cdot 4\pi \\
= 8\pi \sin(4\pi x + 1) \cos(4\pi x + 1) \\
\]
Problem 2: (20 points) Use the limit definition to find the derivative of \( f(x) = x - x^2 \).

\[
\begin{align*}
  f'(x) &= \lim_{{h \to 0}} \frac{[(x+h) - (x+h)^2] - (x-x^2)}{h} \\
  &= \lim_{{h \to 0}} \frac{h - (x^2 + 2xh + h^2) - x + x^2}{h} \\
  &= \lim_{{h \to 0}} \frac{h - x^2 - 2xh - h^2 + x^2}{h} \\
  &= \lim_{{h \to 0}} \frac{h(1 - 2x - h^2)}{h} = 1 - 2x
\end{align*}
\]

Problem 3: (10 points) Find the equation of the tangent line to \( f(x) = 3x^3 + 2x \) when \( x = 1 \).

\[
\begin{align*}
  f'(x) &= 9x^2 + 2 \\
  m &= f'(1) = 11 \\
  \text{point} \quad x &= 1 \\
  y - f(1) &= 5
\end{align*}
\]

\[
\begin{align*}
  y - 5 &= 11(x - 1) \\
  y - 5 &= 11x - 11 \\
  y &= 11x - 6
\end{align*}
\]
Problem 4: (20 points) Let \( x^2 + xy + y^2 = 6 \).

a) Find \( \frac{dy}{dx} \). Show your work.

\[
2x + (x \frac{dy}{dx} + y) + 2y \frac{dy}{dx} = 0
\]

\[
x \frac{dy}{dx} + 2y \frac{dy}{dx} = -2x - y
\]

\[
\frac{dy}{dx} = \frac{-2x - y}{x + 2y} = \frac{-2x - y}{x + 2y}
\]

b) Find the point(s), if any, of the horizontal tangent lines.

Horizontal? \(-2x - y = 0\) \(-y = 2x\) \(y = -2x\) except \(x = y\).

So any points where \(y = 2x\) will result in a horizontal tangent line, except \((0,0)\).

Problem 5: (20 points) Sand is falling off a conveyor belt onto a conical pile at the rate of 15 cubic feet per minute. The diameter of the base of the cone is approximately twice the altitude. At what rate is the height of the pile changing when it is 10 feet high? Note: The volume of a cone is \(V = \frac{1}{3}\pi r^2 h\).

\[
\frac{dV}{dt} = 15 \text{ ft}^3/\text{min}
\]

\[
d = 2h
\]

\[
2r = 2h
\]

\[
r = h
\]

Find \(\frac{dh}{dt}\)

\[
\frac{dV}{dt} = \frac{1}{3} \pi (3h^2) \frac{dh}{dt}
\]

\[
15 = \pi (10)^2 \frac{dh}{dt}
\]

\[
\frac{dh}{dt} = \frac{15}{100\pi} \approx 0.0477 \text{ ft/min}
\]