Problem 1: (10 points) Analytically find the limit, if it exists. Show all of your work.

a) \[
\lim_{x \to 0} \frac{\cos x \tan x}{x}
\]

b) \[
\lim_{x \to 0} \frac{\sqrt{x + 4} - 2}{x}
\]

c) \[
\lim_{x \to 5} \frac{x^2 - 3x - 10}{x - 5}
\]
Problem 2: ( points) Analytically find the limit, if it exists. Show all of your work.

a) \[ \lim_{x \to 2} \sqrt{3x - 6} \]

b) \[ \lim_{x \to 3} \frac{x - 3}{|x - 3|} \]

c) \[ \lim_{x \to \pi} \frac{x}{1 - \cos x} \]
Problem 3: (points) Use the $\varepsilon - \delta$ definition of limit to prove that $\lim_{x \to 2} 3x + 3 = 9$.

Problem 4: (points) Let $f(x) = x^2 + 6$ and $g(x) = \frac{1}{x - 3}$.

a) Find $g(f(x))$

b) Find all values for which $g(f(x))$ is discontinuous.

c) Classify each of the discontinuities in part b as removable or nonremovable.

d) Find and simplify: $\frac{f(x + h) - f(x)}{h}$
Problem 5: (points) Find the limit, if it exists. Show your work. Justify your answer.

a) \[ \lim_{x \to 3^-} \frac{1}{x - 3} \]

b) \[ \lim_{x \to 0^+} (6 + \frac{9}{x^2}) \]

Extra Credit (5 points) Suppose \( 7 - x^2 \leq f(x) \leq 7 + x^2 \) and \( c = 0 \). Find \( \lim_{x \to 0} f(x) \). You must show your work and justify your answer.