There are 5 problems on this exam, worth a total of 100 points. You must show all of your work for credit. No TI-89's or other calculators capable of symbolic manipulations allowed!

**Problem 1:** (30 points) Find the derivative of each function. Pick any 3 out of the 4 to do. Circle the three you want me to grade. If you do not circle the problems you want graded, I will grade parts a, b and c.

a) \( g(x) = \log_8(5x^3 - 6) \)

b) \( f(x) = \ln\left(\frac{6x}{x^2 + 8}\right) \)

c) \( h(x) = 4e^{3x} + 3^{3x} \)

d) \( f(x) = e^{\tan(x)} \)
Problem 2: (30 points) Evaluate the integral. Pick any 3 out of the 4 to do. Circle the three you want me to grade. If you do not circle the ones you want graded, I will grade parts a, b, and c.

a) \[ \int \frac{1 - \cos(x)}{x - \sin(x)} \, dx \]

b) \[ \int \frac{e^{\frac{x}{3}}}{x^2} \, dx \]

c) \[ \int \frac{2x + 1}{x + 1} \, dx \]

d) \[ \int_{2}^{e+1} \frac{1}{x - 1} \, dx \]
Problem 3: (20 points) Let \( f(x) = \sqrt{4x + 1} \)

a) What are the domain and range for \( f(x) \)?

b) Is \( f(x) \) a 1-1 function? Why or why not?

c) Find \( f^{-1}(x) \). State its domain and range.

d) Analytically verify that \( f(x) \) and \( f^{-1}(x) \) are inverses.
Problem 4: (10 points) Pick either part a or part b to do. You only need to do one. Circle the one you want graded.

a) Let \( y = x^{x+1} \). Find \( \frac{dy}{dx} \).

b) Find \( \frac{dy}{dx} \) if \( \ln(xy) = x + y \).

Problem 5: (10 points) A certain type of bacteria increases continuously at a rate proportional to the number present. If there are 500 present initially, and 1000 present 2 hours later, answer each of the following questions. Show your work.

a) Write the differential equation that models this situation.

b) Solve the differential equation to find the function \( P(t) \), which gives the bacterial population when the input is time (in hours).

c) How many bacteria will be present 5.83 hours later?