Problem 1: (points) Let \( p_1 = 1 + 2t, p_2 = 3 + 6t^2, p_3 = 1 + 3t + 4t^2 \).

a) Use coordinate vectors to determine if the given polynomials are linearly independent in \( P_2 \). Recall: the standard basis for \( P_2 \) is \( \{1, t, t^2\} \). Show how you set up the problem. You may use your calculator to do any necessary row reductions.

b) Does the set \( \{p_1, p_2, p_3\} \) form a basis for \( P_2 \)? Why or why not? Justify your answer!
Problem 2 (points) Let $A = \begin{bmatrix} 1 & 3 & -4 & 0 & 1 \\ 2 & 4 & -5 & 3 & -1 \\ 1 & -5 & 0 & -3 & 2 \\ -3 & -1 & 8 & 3 & -4 \end{bmatrix}$.

a) Find a basis for Row(A). You may use a calculator to do any row reductions but explain to me where you got the basis vectors.

b) Find a basis for Col(A). You may use a calculator to do any row reductions but explain to me where you got the basis vectors.

c) Find a basis for Nul(A).
Problem 3 (12 points) Suppose basis $B$ has basis vectors
\[ b_1 = \begin{bmatrix} -1 \\ -1 \end{bmatrix}, \quad b_2 = \begin{bmatrix} -5 \\ -1 \end{bmatrix}, \]
and basis $C$ has vectors
\[ c_1 = \begin{bmatrix} 1 \\ 3 \end{bmatrix}, \quad c_2 = \begin{bmatrix} -4 \\ -10 \end{bmatrix}, \]
for the vector space $\mathbb{R}^2$.

a) Find the change of coordinates matrix from base $B$ to base $C$.

b) Suppose $[x]_B = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$. Find $[x]_C$. 
Problem 4 (points): Suppose a nonhomogeneous system of 15 linear equations in 17 unknowns has a solution for all possible constants on the right side of the equation. Is it possible to find 4 nonzero solutions of the associated homogeneous system that are linearly independent? Explain.

Problem 5 (points) Determine if the set $H$ of all polynomials of the form $a + bt^2$ is a subspace of $P_2$. Justify your answer completely for credit.
Problem 6 ( points) Answer each of the following.

a) If $A$ is $5 \times 7$ what is the smallest possible dimension of $\text{Nul}(A)$?

b) If $A$ is $4 \times 9$ what is the largest possible dimension of $\text{Row}(A)$?

c) True or False. The Column Space of an $m \times n$ matrix is in $\mathbb{R}^m$.

d) True or False. $\mathbb{R}^2$ is a subspace of $\mathbb{R}^4$.

e) True or False: If a finite set $S$ of nonzero vectors spans a vector space $V$, then some subset of $S$ is a basis for $V$.

Problem 7: ( points) Let $H = \{ \begin{bmatrix} 3a + 6b - c \\ 6a - 2b - 2c \\ -9a + 5b + 3c \\ -3a + b + c \end{bmatrix} : a, b, c \in \mathbb{R} \}$.

a) Why is $H$ a subspace? What vector space is it a subspace of?

b) Find a basis for $H$. 
c) What is the dimension of $H$?