Goals: To evaluate a trig function of any real number.

<table>
<thead>
<tr>
<th>θ</th>
<th>sinθ</th>
<th>cosθ</th>
<th>tanθ</th>
</tr>
</thead>
<tbody>
<tr>
<td>30° = π/6</td>
<td>1/2</td>
<td>√3/2</td>
<td>√3</td>
</tr>
<tr>
<td>45° = π/4</td>
<td>√2/2</td>
<td>√2</td>
<td>1</td>
</tr>
<tr>
<td>60° = π/3</td>
<td>√3</td>
<td>1/2</td>
<td>√3</td>
</tr>
</tbody>
</table>

Note: Applications of periodic functions include...
1. Spring vibrations
2. Tides (water depth at a location)
3. Outside temperature throughout the day
4. AC current

Big Idea: We can use trigonometric functions of real numbers to model repetitive phenomena.

The Wrapping Function

Recall radian measure: \( \Theta = \frac{s}{r} \)

So, on the unit circle: \( \Theta = s \)
Section 5.4 Part 2 Trig Functions of Real Numbers Page 2

unit circle: \( x^2 + y^2 = 1 \)

wrapping form: \( W(t) = (x, y) \)

\[ W(0) = (1, 0) \]
\[ W\left(\frac{\pi}{2}\right) = (0, 1) \]
\[ W(\pi) = (-1, 0) \]
\[ W\left(\frac{3\pi}{2}\right) = (0, -1) \]
\[ W(2\pi) = (1, 0) \]

**Ex** Find \( W(t) \) for

\[ t = \frac{4\pi}{3} \]
\[
\cos t = \frac{x}{1} = x \implies \cos \frac{4\pi}{3} = x
\]

\[
x = \cos \frac{4\pi}{3} = -\cos \frac{\pi}{3} = -\frac{1}{2}
\]

\[
\sin t = \frac{y}{1} \implies \sin \frac{4\pi}{3} = y
\]

\[
y = \sin \left(\frac{4\pi}{3}\right) = -\sin \frac{\pi}{3} = -\frac{\sqrt{3}}{2}
\]

\[
W\left(\frac{4\pi}{3}\right) = \left(-\frac{1}{2}, -\frac{\sqrt{3}}{2}\right)
\]

**Def: Trig funs of a real \( t \)**

Let \( W(t) = (x, y) \), where \((x, y)\) on unit circle. Then...

**Exercise:**

In Section 5.4 Part 2 of the textbook, the content discusses trigonometric functions of real numbers, specifically focusing on the unit circle and how to determine the coordinates \((x, y)\) for given angles. The example provided illustrates the process of calculating \( x \) and \( y \) for an angle \( \frac{4\pi}{3} \), resulting in the point \((-\frac{1}{2}, -\frac{\sqrt{3}}{2})\) on the unit circle.
Ex. Show cosine is an even function.

\[
\begin{align*}
\sin t &= y \\
\cos t &= x \\
\tan t &= \frac{y}{x} \\
\cot t &= \frac{x}{y} \\
\sec t &= \frac{1}{x} \\
\csc t &= \frac{1}{y}
\end{align*}
\]

Let \( y = f(x) \).

**even function**: \( f(-x) = f(x) \)

**odd function**: \( f(-x) = -f(x) \)

**Note**: cosine and secant are even. The rest are odd.

\( \text{(ex) } \cos(-x) = \cos x \)

\( \sin(-x) = -\sin x \)
$\cos(-t) = x$

$\cos(-t) = \cos(t)$

So, $\cos t$ is even

---

**Note:** A function is periodic if there is a smallest number $p$ such that $f(t+p) = f(t)$

1. The period of sine, cosine, secant and cosecant is $2\pi$
2. The period of tangent and co-tangent $\pi$.

---

**Ex:** Is it even, odd, or neither?

1. $f(x) = \frac{\cos x}{\tan x \sin x}$

   \[
   f(-x) = \frac{\cos(-x)}{\tan(-x) \sin(-x)} = \frac{\cos x}{\tan x \sin x}
   \]

   $= \frac{\cos x}{1 - \sin^2 x}$
\((-\tan x)(-\sin x)\)
\[= \frac{\cos x}{\tan x \sin x}\]
\[= f(x)\]
\[f \text{ is even}\]

\[b) \quad g(x) = \frac{\sec(x)}{\tan(x)}\]
\[g(-x) = \frac{\sec(-x)}{\tan(-x)}\]
\[= \frac{-\sec x}{-\tan x}\]
\[= -\frac{\sec x}{\tan x}\]
\[= -g(x)\]
\[\text{So, } g \text{ is odd.}\]

\[\text{Fundamental Identities}\]
\[\sin x = \frac{1}{\csc x}, \quad \cos x = \frac{1}{\sec x}, \quad \tan x = \frac{1}{\cot x}\]
\[\tan x = \frac{\sin x}{\cos x}, \quad \cot x = \frac{\cos x}{\sin x}\]
\[\cos^2 x + \sin^2 x = 1\]
\[\cos^2 x = 1 - \sin^2 x\]
Write as a single trig funct.

\[
\frac{1 - \sin^2 \theta}{\cos^2 \theta} = \frac{\cos^2 \theta}{\cos^2 \theta} = \frac{\cos^2 \theta}{\sin^2 \theta} = \frac{1}{\cos^2 \theta} \cdot \frac{\sin^2 \theta}{\cos^2 \theta} = \sin^2 \theta
\]