Section 5.4: Radian Angle Measure and Trigonometric functions of Real Numbers

Goals:

1. To convert between radian and degrees.
2. To compute arc length of a sector of a circle
3. To solve apps.
4. To evaluate a trig function of any real number.

Radian Measure

Radian measure of $\theta$ is given by $\theta = \frac{s}{r}$.

Notes:

- An angle that measures $1$ radian is subtended by an arc length of $1$ radius.
- $\pi$ radians in $1$ revolution is $180^\circ$. 
given by \( \theta = \frac{s}{r} = \frac{2\pi r}{r} = 2\pi \).

So, \( 360^\circ = 2\pi \) radians or

\[ 180^\circ = \pi \text{ radians} \]

\[ s = r\theta \leftarrow \text{arc length formula} \]

\[ \textbf{ex} \] convert to radians or degrees.

\[ \text{a) } -270^\circ \]

\[ -270^\circ \cdot \frac{\pi}{180} = -\frac{27\pi}{18} = -\frac{3\pi}{2} \]

\[ \text{b) } 427^\circ \text{ (approximate)} \]

\[ 427^\circ \cdot \frac{\pi}{180^\circ} = \frac{427\pi}{180} \approx 7.45 \]

\[ \text{c) } \frac{9\pi}{2} \text{ (Assume to be radians when} \]
A wheel is rotating at 200 rpm. Find the angular velocity in radians per second.

\[ \frac{9\pi}{2} \cdot \frac{180^\circ}{\pi} = 810^\circ \]

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Ex. Find the complement of \( \frac{\pi}{3} \)

Complement: \( \frac{\pi}{2} - \frac{\pi}{3} = \frac{3\pi}{6} - \frac{2\pi}{6} = \frac{\pi}{6} \)

Supplement: \( \pi - \frac{\pi}{3} = \frac{3\pi}{3} - \frac{\pi}{3} = \frac{2\pi}{3} \)

Def: Angular Velocity (change in \( \theta \) over time)

\( \frac{\text{angular velocity}}{\text{time}} = \omega = \frac{\theta}{t} \)

"omega"

Ex. A wheel is rotating at 200 rpm. Find the angular velocity in radians per second.
Ex. A truck has a tire of radius 45 cm rotating at 500 rpm. Find the speed of the truck.

\[
\frac{200 \text{ rev}}{1 \text{ min}} \cdot \frac{2\pi \text{ radians}}{1 \text{ rev}} \cdot \frac{1 \text{ min}}{60 \text{ sec}} = 20.94 \frac{\text{ radians}}{\text{ sec}}
\]

\[
\frac{500 \text{ rev}}{1 \text{ min}} \cdot \frac{2\pi (45) \text{ cm}}{1 \text{ rev}} \cdot \frac{1 \text{ Km}}{100000 \text{ cm}} \cdot \frac{60 \text{ min}}{1 \text{ hr}} = 84.8 \frac{\text{ km}}{\text{ hr}}
\]