Goals:

1. To solve a linear system of two equations, two unknowns (6.1)
2. To solve a non-linear system of two equations, two unknowns (6.3).

\[ \begin{align*}
\text{(a)} & \quad \begin{cases}
-3x + 7y = 14 \\
2x - y = -13
\end{cases} \\
\text{solve using substitution} \\
\implies & \quad y = 2x + 13 \\
& \quad (-3x + 7(2x + 13)) = 14 \\
& \quad -3x + 14x + 91 = 14 \\
& \quad 11x = -77 \\
& \quad x = -7 \\
& \quad y = 2(-7) + 13 = -1
\end{align*} \]

\[ \text{intersect pt. } (-7, -1) \]

\[ \text{(b) Same system, use elimination to solve} \]

\[ \begin{align*}
& \quad \begin{cases}
-3x + 7y = 14 \\
7(2x - y = -13)
\end{cases} \\
& \quad \begin{cases}
14x - 7y = -91 \\
-3x + 7y = 14
\end{cases} \\
& \quad \begin{align*}
11x & = -77 \\
x & = -7
\end{align*} \\
& \quad -14 - y = -13
\end{align*} \]
c) solve

\[
\begin{align*}
&y = 2x - 7 \\
&4x - 2y = 14
\end{align*}
\]

\[4x - 2(2x - 7) = 14\]
\[4x - 4x + 14 = 14\]

14 = 14  True  \rightarrow Dependent system

How to write the solutions for Dependent System

Set builder notation
\[\{(x, y) \mid y = 2x - 7\}\]

Ordered pair
\[(x, 2x - 7) \rightarrow (c, 2c - 7)\]

3 situations

1. False statement
2. No solution
Section 9.1, 9.3 Solving Systems of Two Equations, Two Unknowns

- System is consistent and independent
- Parallel lines
  - No solution
  - Consistent system

- Lines overlap.
  - Every ordered pair is a solution.
  - System is consistent and dependent

9.3 Equations are not necessarily linear. (Solutions are still intersection points.)

Example:

\[ y = x^2 + 2x - 3 \]
\[ y = x - 1 \]

Use substitution:

\[ x - 1 = x^2 + 2x - 3 \]
\[ 0 = x^2 + x - 2 \]
\[ 0 = (x + 2)(x - 1) \]
\[ x = -2 \text{ or } x = 1 \]

You can 0, 1, or 2 solutions.
\[ y = -2 - 1 \]
\[ y = -2 - 3 \]
\[ (-2, -3) \quad (1, 0) \]

b) \[ 3x^2 - 2y^2 = 19 \] \[ -2(x^2 - y^2 = 5) \]
\[ -2x^2 + 2y^2 = -10 \]
\[ 3x^2 - y^2 = 19 \]
\[ x^2 = 9 \]
\[ x = \pm 3 \]
\[ (\pm 3, -2)^2 - y^2 = 5 \]
\[ 9 - y^2 = 5 \]
\[ y^2 = 4 \]
\[ y = \pm 2 \]
\[ (\pm 3, -2), (\pm 3, 2) \]

c) \[ (x + 2)^2 + (y - 2)^2 = 13 \]
\[ 2x + y = 6 \]
\[ y = 6 - 2x \]
\[(x+2)^2 + ((6-2x)-2)^2 = 13\]
\[(x+2)^2 + (4-2x)^2 = 13\]
\[x^2 + y^2 + 4 + 16 - 16x + 4y^2 = 13\]
\[5x^2 - 12x + 20 = 13\]
\[5x^2 - 12x + 7 = 0\]
\[(5x - 7)(x - 1) = 0\]
\[5x - 7 = 0 \text{ or } x - 1 = 0\]
\[x = \frac{7}{5} \text{ or } x = 1\]

\[y = 6 - 2\left(\frac{7}{5}\right)\]
\[y = 6 - \frac{14}{5}\]
\[y = \frac{30 - 14}{5}\]
\[y = \frac{16}{5}\]

\((\frac{2}{5}, \frac{16}{5}) \text{ or } (1, 4)\)

\[\text{absolute value} \quad |a|, \ a \geq 0\]
<--- called a piece-wise defined function
\[ |a| = \begin{cases} \ a, & a \geq 0 \\ -a, & a < 0 \end{cases} \] 

\text{Def} 

\text{(Ex) Represent the distance between } x \text{ and 10 using abs. value.} 

\[ |x-10| \text{ same as } |10-x| \]