Goals:
1. To find the length of a missing side of a right triangle.
2. To find the supplement and complement of a given angle.
3. To find a coterminal angle.
4. To set up a function involving the Pythagorean Theorem.

The Pythagorean Theorem

Given: \[ a \quad b \quad c \]

Conclusion: \[ a^2 + b^2 = c^2 \]

Proof: (by President Garfield)

Note that the figure is a trapezoid (why?). Its area is the sum of the component triangles, as well as the formula for area of a trapezoid:

\[
\frac{1}{2}ab + \frac{1}{2}as + \frac{1}{2}cs = \frac{1}{2} (a+b)(a+b)
\]

\[
\frac{1}{2}ab + \frac{1}{2}c^2 = \frac{1}{2}a^2 + ab + \frac{1}{2}b^2
\]

\[
c^2 = a^2 + b^2
\]
Ex. A TV has an aspect ratio of 16 inches by 9 inches and a 52 inch diagonal. Find the dimensions.

\[ x^2 + y^2 = 52^2 \]
\[ \frac{x}{y} = \frac{16}{9} \]
\[ x = \frac{16}{9} y \]
\[ \left(\frac{16}{9} y\right)^2 + y^2 = 52^2 \]
\[ \frac{256}{81} y^2 + y^2 = 52^2 \]
\[ \frac{256 + 81}{81} y^2 = 2704 \]
\[ \frac{337}{81} y^2 = 2704 \]
\[ y^2 = 2704 \cdot \frac{81}{337} \]
\[ y^2 = \frac{219024}{337} \]
\[ y = \sqrt{\frac{219024}{337}} \text{ inches} \]
\[ \approx 25.5 \text{ inches} \]
\[ x = \frac{16}{9} \sqrt{219024} \text{ inches} \]
Ex. A bridge that is 5000 feet long expands 1 foot in the summer heat. For simplicity, assume the bridge bows up in the middle. Use the Pythagorean Theorem to estimate the height of the bridge midway across it.

The bridge bulges up about 50 feet!
a ray about its endpoint.

2. 1 degree is $\frac{1}{360}$ of a full rotation.
3. cotermin al angles share the same terminal side.

Ex) Find a coter minal angle for $190^\circ$ that is between $0^\circ$ and $360^\circ$.

$$790^\circ - 360^\circ - 360^\circ = 70^\circ$$

Ex) Find the exact length of the missing side.

\[ \triangle \]
Two ships leave the same port at the same time, one is traveling north at 10 mph, and the other goes east at 12 mph.

Note: In a 30°-60°-90° Triangle, the sides are...

![30°-60°-90° Triangle](diagram.png)

- a
- \(\sqrt{3}a\)
- 2a

Def: A function is an equation of the form

\[ y = \text{(some formula in terms of } x) \] as long as any particular \( x \)-value yields a single \( y \)-value.

\[ a^2 + b^2 = (2a)^2 \]

\[ b^2 = 4a^2 - a^2 \]

\[ b = \sqrt{3}a \]
Write a function that gives the distance between the ships after $x$ hours.

\[ \text{Distance} = (\text{Rate}) \cdot (\text{Time}) \]

\[ d^2 = (10x)^2 + (12x)^2 \]
\[ d^2 = 100x^2 + 144x^2 \]
\[ d^2 = 244x^2 \]
\[ d = \sqrt{244}x \]
\[ d = 2\sqrt{61}x \]

Using function notation,
\[ d(x) = 2\sqrt{61}x \quad \text{(in miles)} \]