Goals:
1. To graph VVF's
2. To generalize the ideas of limits and continuity to VVF's.

Recall: Parametric Equations $x = f(t), y = g(t)$ can describe a curve in the plane. We graph these values to create a vector-valued function:

$$\vec{r}(t) = \langle f(t), g(t) \rangle = f(t) \hat{i} + g(t) \hat{j}$$

Sketch the graph given by...

$$\vec{r}(t) = (1-t) \hat{i} + \sqrt{t} \hat{j}$$

Domain: $t \geq 0$

In calc lingo...

- parametric eqns.
- $t$ is the parameter

- $x = 1-t$
- $y = \sqrt{t}$

- inputs

- outputs
Eliminate the parameter to get a cartesian equation.

\[ x = 1 - t \quad \rightarrow \quad x = 1 - x \]
\[ y = \sqrt{x} \]
\[ y = \sqrt{1 - x} \]

b) \( \vec{r}(t) = \left< 3 \cos t, 4 \sin t, \frac{t}{2} \right> \)

**Domain (allowable input values)**

\[ \rightarrow \quad x = 3 \cos t \]
\[ \rightarrow \quad y = 4 \sin t \]
\[ \rightarrow \quad z = \frac{t}{2} \]

\[ \text{parametric eqns.} \]

Domain is all Reals: \((-\infty, \infty)\)
Note: This curve is called an elliptical helix. It lies on the cylinder given

\[ x = 3 \cos t, \quad y = 4 \sin t, \quad z = \frac{t}{4} \]

\[ \cos^2 t + \sin^2 t = 1 \]

\[ \left(\frac{t}{3}\right)^2 + \left(\frac{y}{4}\right)^2 = 1 \]

\[ \frac{x^2}{9} + \frac{y^2}{16} = 1 \]

elliptical cylinder in space

Sketch the curve of intersection of

\[ z = x^2 + y^2 \quad \text{and} \quad z = 4 \]

Then write the curve as a vector-valued function.

\[ y = \sqrt{x^2 + y^2} \quad \sqrt{x^2 + y^2} = 4 \]
\[ \cos^2 t + \sin^2 t = 1 \]

\[ x^2 + y^2 = 4 \]
\[ \frac{x^2}{4} + \frac{y^2}{4} = 1 \]
\[ \left( \frac{x}{2} \right)^2 + \left( \frac{y}{2} \right)^2 = 1 \]

\[ \frac{x}{2} = \cos t, \quad \frac{y}{2} = \sin t \]

\[ x = 2 \cos t, \quad y = 2 \sin t, \quad z = 4 \]

\[ \vec{r}(t) = \langle 2 \cos t, 2 \sin t, 4 \rangle \]

**Notes**

1. The limit of \( \vec{r}(t) \) is the limit of each of its components.

2. \( \vec{r}(t) \) is continuous iff each of its component functions are continuous. In symbols...
\[
\lim \vec{r}(x) = \lim_{x \to a} \langle x(t), y(t), z(t) \rangle \\
= \langle x(a), y(a), z(a) \rangle
\]

\[y = f(x)\]

\[
\lim_{x \to a} f(x) = f(a)
\]

\[\text{(Ex)}\]

a) Evaluate

\[
\lim_{t \to 0} \langle \frac{e^t - 1}{t}, \sin t, \frac{t^2 - t}{t} \rangle
\]

by def: \[
= \langle \lim_{t \to 0} \frac{e^t - 1}{t}, \lim_{t \to 0} \sin t, \lim_{t \to 0} \frac{t^2 - t}{t} \rangle
\]

= \[
= \langle \lim_{t \to 0} \frac{e^t}{1}, 0, \lim_{t \to 0} \frac{t^2 - t}{t} \rangle
\]

= \[
= \langle 1, 0, -1 \rangle
\]

b) On which interval is \( \vec{r}(x) = \sqrt{x} \vec{i} + \sqrt{x-1} \vec{j} \) continuous?

Note: radical functions are continuous on their domains.
\( e^x \) Show \( \mathbf{r}(t) = e^{-t} \cos 10t \mathbf{i} + e^{-t} \sin 10t \mathbf{j} + e^{-t} \mathbf{k} \) lies on the cone \( x^2 + y^2 = z^2 \)

\[
\begin{align*}
x &= e^{-t} \cos 10t, \\
y &= e^{-t} \sin 10t, \\
z &= e^{-t}
\end{align*}
\]

\[
(e^{-t} \cos 10t)^2 + (e^{-t} \sin 10t)^2 = (e^{-t})^2
\]

\[
e^{-2t} \cos^2 10t + e^{-2t} \sin^2 10t = e^{-2t}
\]

\[
e^{-2t} \left( \cos^2 10t + \sin^2 10t \right)
\]

\[
e^{-2t} \cdot 1
\]

\[
e^{-2t} \checkmark
\]

This does it.