**Goal:** To set up and evaluate triple integrals.

In general, the triple integral over a region $Q$, is given by...

$$
\iiint_{Q} f(x,y,z) \, dV
$$

Note: To find the limits of integration, first find the inner limits then project $Q$ onto
Note: To find the limits of integration, first find the inner limits, then project $Q$ onto the axis associated with the outer two variables of integration and proceed as you would for a double integral.

**Example** Find the volume between the paraboloids $Z = x^2 + y^2$, $Z = 4 - x^2 - y^2$ using a triple integral.

\[
Z = 4 - (x^2 + y^2)
\]

\[
\iiint_{Q} 1 \, dz \, dy \, dx
\]

\[
= \int_{-\sqrt{2}}^{\sqrt{2}} \int_{-\sqrt{2-y^2}}^{\sqrt{2-y^2}} \int_{x^2+y^2}^{4-x^2-y^2} dz \, dy \, dx
\]

\[
= \iint_{Q} (4 - x^2 - y^2 - x^2 - y^2) \, dy \, dx
\]

\[
= \iint_{Q} (4 - 2x^2 - 2y^2) \, dy \, dx
\]
\[
\iint (y - 2x^2 - 2y^2) \, dy \, dx
\]
\[
\iiint f(x,y) \, dA
\]
\[
\iiint (y^2 - 2(2x+y^2)) \, dV
\]
\[
\iiint (y - 2r^2) \, r \, dr \, d\theta
\]
\[
(y \pi)
\]

Express \( \iiint f(x,y,z) \, dV \) as an iterated integral in 6 different ways.

Q: Bounds are \( x^2 + z^2 = 4 \), \( y = 0 \), \( y = 6 \)

\[
\iiint f \, dz \, dy \, dx
\]
\[
\iiint f \, dy \, dz \, dx
\]
\[
\iiint f \, dx \, dy \, dz
\]

\[
\iiint h_2(x,y) \, dz
\]
\[
\iiint h_1(x,y) \, dy
\]

\[
\iiint h_3(y,z) \, dz
\]
\[
\iiint h_4(y,z) \, dy
\]

\[
\iiint h_5(x,y) \, dx
\]
\[
\iiint h_6(x,y) \, dy
\]
Evaluate \( \int \int \) \( y \, dV \), where \( Q \) is bounded by the paraboloid \( y = 3x^2 + 3z^2 \) and \( y = 3 \).
\[ \int_{\frac{\pi}{2}}^{\pi} \int_{0}^{1} (9 - 9r^2) r \, dr \, d\theta \]
\[ = \frac{9}{2} \int_{0}^{1} \left( r - r^5 \right) dr \, d\theta \]
\[ = \frac{9}{2} \int_{0}^{\frac{\pi}{2}} 1 \, d\theta \int_{0}^{1} (r - r^5) \, dr \]
\[ = \frac{9}{\pi} \cdot 2\pi \left[ \frac{r^2}{2} - \frac{r^6}{6} \right]_0^1 \]
\[ = 9\pi \left[ \frac{1}{2} - \frac{1}{6} \right] \]
\[ = 9\pi \frac{12 - 6}{12} \]
\[ = \frac{9\pi \cdot 6}{12} \]
\[ = \frac{3\pi}{2} \]