Section 16.4: Green's Theorem

Goal: To use Green's Theorem to evaluate a line integral

Green's theorem relates line integrals to double integrals.

Recall: A curve $C$ in a plane is positively oriented if it is traveled in a counterclockwise direction.

Theorem 1: Green's Theorem

Let $C$ be a positively oriented, piecewise smooth, simple closed curve that bounds a region $R$ in the $xy$-plane. Then, as long as $P$ and $Q$ have continuous partial derivatives in an open region containing $R$,

$$
\int_C P \, dx + Q \, dy = \iint_R \left( \frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) \, dA
$$

Theorem 2: Corollary to Green's Theorem

If $R$ is a plane region bounded by a piecewise smooth simple closed curve $C$, oriented counterclockwise, then the area of $R$ is given by

$$
A = \frac{1}{2} \int_C xy \, dy - ydx
$$