Homework Section 12.5

1. Let \( \mathbf{u} = \langle -1, 2, -1 \rangle \) and \( \mathbf{v} = \langle 2, 1, 1 \rangle \). Find \( \mathbf{u} \times \mathbf{v} \) and verify that it is orthogonal to both \( \mathbf{u} \) and \( \mathbf{v} \).

2. Let \( \mathbf{v} = 2\mathbf{i} + \mathbf{j} + \mathbf{k} \) and \( \mathbf{w} = 3\mathbf{i} - \mathbf{k} \). Find two vectors that are orthogonal to both \( \mathbf{v} \) and \( \mathbf{w} \).

3. Let \( \mathbf{u} = 3\mathbf{i} - \mathbf{k} \) and \( \mathbf{v} = 2\mathbf{j} + \mathbf{k} \). Find \( \mathbf{u} \times \mathbf{v} \) and graph \( \mathbf{u} \), \( \mathbf{v} \), and \( \mathbf{u} \times \mathbf{v} \) in component form.

4. In the following picture, is \( \mathbf{u} \times \mathbf{v} \) directed towards you or away from you?

5. Suppose \( |\mathbf{u}| = 3 \), \( |\mathbf{v}| = 12 \), and the angle between \( \mathbf{u} \) and \( \mathbf{v} \) is \( \pi/6 \) radian. Find \( |\mathbf{u} \times \mathbf{v}| \).

6. Use the cross product to determine the area of a parallelogram that has vertices \((2, 0), (4, 2), (6, 1), (4, -1)\).

7. Find the volume of the parallelepiped determined by \( \mathbf{u} = -\mathbf{i} + 2\mathbf{j} - \mathbf{k} \) and \( \mathbf{v} = 2\mathbf{i} + \mathbf{j} + \mathbf{k} \) and \( \mathbf{w} = 3\mathbf{i} - \mathbf{k} \).

8. A ten inch long wrench grips a bolt located at the origin. A force of magnitude 20 pounds is applied to the end of the wrench at an angle of 30 degrees to the handle. Find the magnitude of the torque on the bolt.

9. Refer to the last problem. Find the magnitude of the force needed to apply 420 foot-pounds of torque on the bolt if length of the wrench remains unchanged, but the force is applied in the direction of \( \langle 1, 2, 0 \rangle \). (Hint: Let \( \mathbf{D} \) lie on the positive \( x \)-axis. Note that you are looking for \( |\mathbf{F}| \).)
10. Suppose you want to multiply two matrices $A$ and $B$. Multiplication of matrices is defined so that the element in the $i$th row and $j$th column (denoted $(ab)_{ij}$) of the product, is given by the dot product of the $i$th row of $A$ and $j$th column of $B$. So, for example, let $A = \begin{pmatrix} -2 & 5 \\ 3 & 8 \end{pmatrix}$ and $B = \begin{pmatrix} 2 & -3 & 8 \\ -5 & 1 & 0 \end{pmatrix}$. Then

$$AB = \begin{pmatrix} -2\times2 + 5\times(-5) & (-2)\times(-3) + 5\times1 & -2\times8 + 5\times0 \\ 3\times2 + 8\times(-5) & 3\times(-3) + 8\times1 & 3\times8 + 8\times0 \end{pmatrix} = \begin{pmatrix} -29 & 11 & -16 \\ -34 & -1 & 24 \end{pmatrix}$$

Now, let $C = \begin{pmatrix} -2 & 3 & 8 \\ -5 & 1 & 0 \end{pmatrix}$ and $D = \begin{pmatrix} -1 & 2 \\ 3 & -4 \\ 5 & 6 \end{pmatrix}$. Find both $CD$ and $DC$ by hand.

11. Some observations are in order:

   a) For a product matrix, say $AB$, to be defined, the number of columns in $A$ must be the same as the number of rows in $B$. Note that in the above exercise $AB$ is defined since $A$ has two columns and $B$ has two rows, but $BA$ is undefined since $B$ has three columns and $A$ only has two rows.

   b) Suppose matrix $A$ has order $m \times r$ and matrix $B$ has order $r \times n$. Then the order of the product $AB$ is $m \times n$. In the above exercise, for example, $AB$ has order $2 \times 3$ since $A$ has two rows and $B$ has three columns.

Suppose a matrix $A$ has order $3 \times 4$ and matrix $B$ has order $5 \times 3$. If possible, find the order of the products $AB$ and $BA$. 