Homework Section 14.5

1. Use the Chain Rule to find \( \frac{dz}{dt} \).
   a) \( z = xy^2 - x^2 y, \quad x = 2 + t^3, \quad y = 1 - t^2 \)
   b) \( z = y \ln(2x + y), \quad x = \cos t, \quad y = \sin t \)

2. Use the Chain Rule to find \( \frac{\partial z}{\partial s} \) and \( \frac{\partial z}{\partial t} \)
   a) \( z = \frac{y}{x}, \quad x = se^{-t}, \quad y = 1 + se^t \)
   b) \( z = e^t \sin \theta, \quad r = st, \quad \theta = \sqrt{s^2 + t^2} \)

3. Use the Chain Rule to find \( \frac{\partial z}{\partial v} \) for \( z = \cos x \sin y, \quad x = u^2 + v, \) and \( y = u^2 - v^2 \). Write your answer as a function of \( u \) and \( v \).

4. Let \( z = x^3 + xy^2, \quad x = uv^2 + w^3, \) and \( y = u + ve^w \). Use the Chain Rule to find \( \frac{\partial z}{\partial u} \), \( \frac{\partial z}{\partial v} \), and \( \frac{\partial z}{\partial w} \) when \( u = 1, \quad v = 2, \quad w = 0 \).

5. Let \( z = \ln(u^2 + v^2 + w^2), \quad u = 2x + y, \quad v = x - 2y, \) and \( w = 3xy \). Use the Chain Rule to find \( \frac{\partial z}{\partial x} \) and \( \frac{\partial z}{\partial y} \) when \( x = y = 1 \).

6. The radius of a right circular cone is increasing at a rate of 1.6 in/s while its height is decreasing at a rate of 2.4 in/s. At what rate is the volume of the cone changing when the radius is 110 in. and the height is 130 in.?

7. According to Ohm's law, \( I = \frac{V}{R} \), where \( I \) stands for current (in amps) in an electric circuit, \( V \) is voltage, and \( R \) is resistance (in ohms). As time \( t \) (in seconds) increases, the battery starts to deplete and the voltage decreases while the resistance increases due to heat in the resistor. Find the rate of change of the current with respect to time when \( R = 100 \) ohms, \( dR/dt = 0.5 \) ohms/sec, \( I = 0.1 \) amp, and \( dV/dt = -0.1 \) volt/sec.

8. Use an implicit differentiation formula to find \( \frac{\partial z}{\partial x} \) and \( \frac{\partial z}{\partial y} \):
   a) \( x^3 + y^2 + z^2 = 4xyz \)
   b) \( xyz = \sin(x + y + z) \)