Homework Section 14.6

1. Find the directional derivative of the function at the given point in the direction of the vector $\mathbf{v}$.

   a) $f(x, y) = \frac{y}{x}$, $(-2, 6)$, $\mathbf{v} = \langle -1, 4 \rangle$

   b) $g(s, t) = s^3 e^r$, $(2, 0)$, $\mathbf{v} = i + j$

   c) $f(x, y, z) = \sqrt{x^2 + y^2 + z^2}$, $(1, -2, 2)$, $\mathbf{v} = \langle -2, 3, -6 \rangle$

   d) $g(x, y, z) = z^3 - xy^2$, $(1, 6, 2)$, $\mathbf{v} = 12i + 3j + 4k$

2. Find the directional derivative of $f(x, y) = \sqrt{xy}$ at point $P(4, 9)$ in the direction of point $Q(6, 5)$.

3. Recall that the direction of a planar vector is given by an angle $\theta$ measured counterclockwise from the positive $x$-axis, and a unit vector with direction $\theta$ is given by $\mathbf{u} = \langle \cos \theta, \sin \theta \rangle$. Find the directional derivative of $f$ at the given point in the direction of $\theta$.

   a) $f(x, y) = 2x^2 y^3 + x^3 y$, $(1, -2)$, $\theta = \frac{\pi}{6}$

   b) $f(x, y) = 4ye^{-x}$, $(0, 3)$, $\theta = \frac{\pi}{2}$

4. Find the maximum rate of change of $f$ at the given point and the direction in which it occurs.

   a) $f(x, y) = 2 \cos(xy)$, $(\pi/2, 1)$

   b) $f(x, y, z) = xy^2 z^3$, $(2, 1, 1)$

5. Find the direction in which the function $f(x, y) = x^3 y^2 - xy^4$ decreases the fastest at the point $(3, -1)$.

6. The shape of a hill is modeled by the equation $z = 800 - 0.02x^2 - 0.01y^2$. Suppose a man is on the hill at point $(50, 90, 669)$.

   a) Which way should he go in order to hike in the steepest direction. (I'm looking for a two-dimensional vector here.)

   b) If he hikes in the direction you found in part (a), what would be his angle of inclination as he begins his ascent up the hill?
7. Find the equation of the tangent plane at the given point.
   
   a) \( x^3 = y^2 + z^2 - 7 \), \((-2, 1, 0)\)  
   b) \( 3x^2 - 2y^2 - z^2 + xyz = -5 \), \((1, -1, 2)\)