Homework Section 15.5

1. Calculate the mass and the center of mass of the lamina that occupies the region $R$ and has density function $\rho(x, y)$.

   a) $R = \{(x, y) | -1 \leq x \leq 1, \ 0 \leq y \leq 2\}; \ \rho(x, y) = x^2y$

   b) $R$ is bounded by the parabola $x = y^2$ and the line $y = -x + 2; \ \rho(x, y) = 4$

2. A lamina is given by the quarter disk $x^2 + y^2 \leq 4$ in the first quadrant. Calculate its center of mass when the density at any point on the lamina is proportional to its distance from the origin. [Hint: using polar coordinates, the distance to the origin is $r$.]

3. Find the moments of inertia $I_x, I_y, I_0$ for the lamina in exercise 2.

4. Suppose a lamina corresponds to the region between the curve $y = \cos x$ and the $x$-axis from $x = 0$ to $x = \pi/2$. Given that the lamina has constant density $\rho(x, y) = \rho$. Calculate the moments of inertia $I_x$ and $I_y$ and the radii of gyration $\bar{x}$ and $\bar{y}$. 