Homework Section 16.4

1. Consider the line integral \( \int_C x^2y \, dx + xy^3 \, dy \), where \( C \) is the triangle with vertices (0,0), (1,0), and (1,2):
   a) Evaluate the line integral directly.
   b) Evaluate the line integral using Green’s Theorem (Note how much easier this is!!!).

2. Use Green’s Theorem to evaluate the line integral along \( C \), which is a positively oriented curve:
   a) \( \int_C 2ye \, dx + e^y \, dy \), \( C \) is the square with sides \( x = 0, x = 1, y = 0, \) and \( y = 1 \).
   b) \( \int_C (2y + \cos x^2) \, dx + (x + e^{\sqrt{y}}) \, dy \), \( C \) is the boundary of the region enclosed by the parabolas \( y = x^2 \) and \( x = y^2 \).
   c) \( \int_C (x + y) \, dx + (xy) \, dy \), \( C \) is the boundary of the region lying between the graphs of \( x^2 + y^2 = 1 \) and \( x^2 + y^2 = 4 \).

3. Use Green’s Theorem to find the work done by the force field \( F(x, y) = x^2 \, \mathbf{i} + y(x - y) \, \mathbf{j} \) in moving a particle from (0, 1) along the y-axis to the origin, then along the x-axis to (1,0), and then along the straight line segment back to (0, 1).

4. Use a line integral to find the area of the region bounded by the graphs of \( y = 2x + 1 \) and \( y = 4 - x^2 \).