Show all work for each question, full credit will be given only if all work is shown even when correct answer is circled. If answer is none of these, write correct answer in. Each question is worth 4 points. Good luck!

1. Identify the graph of the parabola and find the coordinates of the vertex:
   \[ f(x) = -(x + 5)^2 - 1 \]
   (A) \[ \text{vertex } (-5, -1) \]
   (B) \[ \text{vertex } (-5, 1) \]
   (C) \[ \text{vertex } (-1, 5) \]
   (D) \[ \text{vertex } (5, 1) \]
   (E) None of these

2. Identify the quadratic function written in standard form and find the vertex of the graph:
   \[ f(x) = -8x^2 + 3 + 2x^2 \]
   a. \[ f(x) = 2(x - 2)^2 + 5 : \text{ Vertex } (5, 2) \]
   b. \[ f(x) = 2(x - 2)^2 + 5 : \text{ Vertex } (2, 5) \]
   c. \[ f(x) = 2(x + 2)^2 + 5 : \text{ Vertex } (-2, 5) \]
   d. \[ f(x) = 2(x + 2)^2 + 5 : \text{ Vertex } (5, -2) \]
   e. None of these:
3. Determine the right-hand and left-hand behavior of the graph of the function:
   \( f(x) = -3x^6 + 5x^4 + 3 \)
   
   a. Up to the left, down to the right
   b. Up to the left and right
   c. Down to the left, up to the right
   d. Down to the left and right
   e. None of these.

4. Find all the real zeros of the function: \( f(x) = x^3 - 6x^2 - 9x + 14 \)
   
   a. \( x = 1, x = 2, x = -7 \)
   b. \( x = 1, x = -2, x = -7 \)
   c. \( x = -1, x = -2, x = -7 \)
   d. \( x = -1, x = 2, x = -7 \)
   e. None of these.

5. Divide using long division: \( (-3x^3 + 2x + 1) \div (x - 3) \)
   
   a. \( -3x^2 - 7x + 21 + \frac{64}{x - 3} \)
   b. \( -3x^2 - 7x - 20 + \frac{60}{x - 3} \)
   c. \( -3x^2 - 9x - 25 + \frac{74}{x - 3} \)
   d. \( -3x^2 + 9x + 29 + \frac{90}{x - 3} \)
   e. None of these.

6. Use synthetic division to determine which one of the following is NOT a factor of \( x^3 - 3x^2 - 16x + 48 \)
   
   a. \( x - 3 \)
   b. \( x + 3 \)
   c. \( x + 4 \)
   d. \( x - 4 \)
   e. All are factors

7. Perform the operation and write results in standard form: \((-9 + 3i) - (-6 - i)\)
   
   a. \(-15 + 2i\)
   b. \(-3 + 2i\)
   c. \(-15 - 2i\)
   d. \(57 = 9i\)
   e. None of these: \(-3 + 4i\)
8. Rationalize the fraction and write results in standard form: \( \frac{-2+4i}{3-9i} \)

\[ \frac{3+9i}{3+9i} \]

\[ = \frac{-6 - 18i + 12i + 36i^2}{9 - 81i^2} + 81 \]

\[ = \frac{-42 - 6i}{90} = \frac{-42}{90} + \frac{-6i}{90} \]

9. Find a polynomial with integer coefficients that has the given zeros: \(-3, 4i, 5i\)

Option A:

\[ f(x) = x^5 + 3x^4 + 41x^3 + 123x^2 + 400x + 1200 \]

\[ = (x+3)(x-4i)(x+4i)(x-5i)(x+5i) \]

\[ = (x+3)(x^2 + 16)(x^2 + 25) \]

\[ = x^5 + 41x^3 + 400x + 3x^4 + 123x^2 + 1200 \]

\[ = x^5 + 3x^4 + 41x^3 + 123x^2 + 400x + 1200 \]

10. Find all the zeros (real or complex) of the function: \( f(x) = x^4 - 6x^3 + 11x^2 + 6x - 12 \)

Try \( 1 - 6 \frac{12}{1 - 5} \)

\[ \frac{12}{1 - 5} \]

Try \(-1 \frac{12}{1 - 6} \)

\[ \frac{12}{1 - 6} \]

\[ x^2 - 6x + 12 \]

Use quadratic formula:

\[ 6 \pm \sqrt{36 - 48} \]

\[ = \frac{6 \pm \sqrt{-12}}{2} \]

\[ = \frac{6 \pm 2i \sqrt{3}}{2} \]

\[ = 3 \pm i \sqrt{3} \]

\[ 1, -1, 3 + i \sqrt{3}, 3 - i \sqrt{3} \]
11. Find the vertical asymptote(s) if any, for \( f(x) = \frac{4x + 6}{x^2 - 10x + 21} \)

- a. \( x = 3, x = 7 \)
- b. \( x = 3, x = 7, x = -2/3 \)
- c. \( x = -2/3 \)
- d. No vertical asymptotes
- e. None of these: ____________

\[(x - 7)(x - 3) = 0 \]
\[x = 7, x = 3\]

12. Identify the horizontal asymptote, if any, for \( f(x) = \frac{8}{x - 5} \)

- a. \( y = \frac{8}{5} \)
- b. \( y = 0 \)
- c. \( y = 8 \)
- d. No horizontal asymptote
- e. None of these: ____________

13. Identify the graph of the rational function: \( f(x) = \frac{x^2}{x^2 - 9} \)

- y = 1
- \( x = 3, -3 \)

C

E. None of these
14. (8 points) Sketch the graphs of the functions. Include the key points and asymptote lines.
   a. \( f(x) = 3^x - 2 \)
   b. \( f(x) = \log(x + 4) \)

\[ 
\begin{align*}
  x &= 4 \\
  4 - x &> 0 \\
  4 &> x \\
  x &< 4 \\
\end{align*}
\]

15. Find the domain, vertical asymptote, and x intercept of the logarithmic function: \( \log_4(4 - x) \).
   a. Intercept: (3, 0), Vertical Asymptote: \( x = 4 \), Domain \((-\infty, 4)\)
   b. Intercept: (-3, 0), Vertical Asymptote: \( x = 4 \), Domain \((-\infty, -4)\)
   c. Intercept: (3, 0), Vertical Asymptote: \( x = 4 \), Domain \((4, \infty)\)
   d. Intercept: (-3, 0), Vertical Asymptote: \( x = -4 \), Domain \((4, \infty)\)

16. Find the expanded form of the logarithm: \( \log_4\sqrt[12]{\frac{x^6 y^2}{z^3}} \)
   a. \( \sqrt[12]{\frac{12 \log x \log y}{3 \log z}} \)
   b. \( 24 \log x + 8 \log y - 12 \log z \)
   c. \( 5/4 \log(x + y + z) \)
   d. \( 3/2 \log x + 1/2 \log y - 3/4 \log z \)
   e. None of these: __________________

17. Which is the logarithm written as a single expression for \( \ln 3x + 3(\ln x - \ln y) \)
   a. \( \ln \frac{6x}{3y} \)
   b. \( \ln \frac{3x^4}{y^3} \)
   c. \( \ln \frac{9x^2}{y} \)
   d. \( \ln 3x^4 y^3 \)
   e. None of these: __________________
18. Given \( \log_2 2 = 0.315 \) and \( \log_2 11 = 1.091 \), find \( \log_2 22 \).

\[
= \log_2 2 + \log_2 11 \\
= 0.315 + 1.091 \\
= 1.406
\]

19. Solve: \( \frac{1}{27} = 9^{5x+4} \)

\[
3^{-3} = 3^{2(5x+4)} \\
-3 = 10x + 8 \\
10x = -11 \\
x = -\frac{11}{10}
\]

20. Solve: \( 7e^{-0.9x} + 32 = 60 \)

\[
7e^{-0.9x} = 28 \\
e^{-0.9x} = 4 \\
-0.9x = \ln 4 \\
x = -\frac{\ln 4}{-0.9}
\]

21. Solve: \( \log_5(x + 4) - \log_5 x = 3 \)

\[
\log_5 \frac{x + 4}{x} = 3 \\
\frac{x + 4}{x} = 5^3 \\
125x = x + 4 \\
124x = 4 \\
x = \frac{4}{124}
\]

22. If \$4500 is invested with an interest rate of 5% compounded continuously, determine the amount of money in the account after 7 years.

\[
A = 4500e^{0.05(7)}
\]

\[
= 6385.80
\]
23. An automobile manufacturer is introducing a new fuel efficient vehicle and estimated the demand for the car as: \( N = 57,000 \ln(5t + 3) \) where \( N \) is the estimated number of cars and \( t \) is the number of years after the car is introduced. When will the demand be 200,000 cars. Round to nearest tenth.

a. 7.3 years
b. 4.6 years
\( \underline{c. 6.1 \text{ years}} \)
d. 6.8 years
e. None of these:

\[
200,000 = 57,000 \ln(5t + 3)
\]
\[
3.5087 + 1.93 = 5t + 3
\]
\[
33.407 = 5t + 3
\]
\[
t = 6.1
\]

24. A rumor begins at a closed campus high school that the President will visit the school the next day. There are 2600 students at the high school and the spread of the rumor is modeled by \( S = \frac{2600}{1 + 2599e^{-0.7t}} \) where \( S \) is the number of students who have heard the rumor and \( t \) is the time in minutes. How long before 70% of the students have heard the rumor? Round to the nearest tenth.

a. 11.8 minutes
b. 14.3 minutes
c. 11.2 minutes
\( \underline{d. 12.4 \text{ minutes}} \)
e. None of these:

\[
0.7 (2600) = \frac{2600}{1 + 2599e^{-0.7t}}
\]
\[
1830 = \frac{2600}{1 + 2599e^{-0.7t}}
\]
\[
1 + 2599e^{-0.7t} = \frac{2600}{1830}
\]
\[
e^{-0.7t} = -8.72856
\]
\[
t = 12.4
\]

25. The number of bacteria \( N \) is a culture is given by \( N = 50e^{kt} \) where \( t \) is time in hours. If \( N = 90 \) when \( t = 6 \), what is the time required for the original population to quadruple in size? Round to two decimal places.

a. 22.72 hours
b. 24.00 hours
c. 10.58 hours
\( \underline{d. 14.15 \text{ hours}} \)
e. None of these:

\[
4 = e^{0.98t}
\]
\[
\ln 4 = 0.98t
\]
\[
t = 14.15
\]