MATH 116
QUIZ 2

Show all work for full credit on any problem. Clearly indicate all answers.

1. Write the standard form of the parabola and state the vertex:
   \[ f(x) = -2x^2 - 4x - 5 \] (2 points)
   \[ -2 \left( x^2 + 2x \right) - 5 \\
   -2 \left( x^2 + 2x + 1 \right) - 5 + 2 \\
   -2(x+1)^2 - 3 \]
   Vertex: \((-1, -3)\)

2. Find the quadratic function that has a minimum at (1, -2) and passes through (0, 0).
   \[ 0 = a(0-1)^2 - 2 \]
   \[ 0 = a(1)^2 - 2 \]
   \[ a = 2 \]

3. Determine the left and right behavior of the graph:
   \[ f(x) = -x^5 + 2x^2 - 1 \]
   a. Up to the left, down to the right
   b. Up to the left and right
   c. Down to the left, up to the right
   d. Down to the left and right
   e. None of these

4. Find all the real zeros by factoring: \( f(x) = x^2 + 3x - 28 \)
   \[ (x + 7)(x - 4) \]
   \[ x = -7 \quad \text{or} \quad x = 4 \]

5. Divide using long division: \((6x^3 + 7x^2 - 10x - 6) \div (2x + 1)\)
   \[ 3x^2 + 2x - 6 \]

6. Use synthetic division to factor the polynomial completely if -2 is a zero: \( f(x) = x^3 - 4x^2 - 7x + 10 \)
   \[ -2 | \begin{array}{rrrr}
   1 & -4 & -7 & 10 \\
   & 2 & 12 & -10 \\
   \hline
   1 & -6 & 5 & 0 \\
   \end{array} \]
   \( x^2 - 6x + 5 \)
   \( (x - 5)(x - 1) \)
7. Use synthetic division to find \( f(-3) \):
\[
\begin{array}{c|cccc}
-3 & 4 & -12 & 36 & 10 \\
 & & 0 & 12 & 39 \\
\hline
 & 4 & -12 & 36 & 104 \\
\end{array}
\]
\[
\frac{4}{-12} \quad \frac{36}{104} \quad \frac{10}{1}
\]

8. List ALL POSSIBLE rational zeros of the function:
\[ f(x) = 2x^3 - 8x^2 + 3x - 6 \]
\[ \pm 1, \pm \frac{1}{2}, \pm 2, \pm 3 \]
\[ \pm \frac{3}{2}, \pm 6 \]

9. Find all the real zeros of the function and graph:
\[ f(x) = 6x^4 + 32x^3 - 70x^2 \] (2 points)

\[ x = 0 \quad (3x - 5)(x + 7) \]
\[ x = \frac{5}{3}, x = -7 \]

10. Find all the real zeros of the function and graph:
\[ f(x) = 2x^3 + 5x^2 - x - 6 \] (2 points)

\[ \text{Try } -2 \]
\[
\begin{array}{c|cccc}
2 & 2 & 5 & -1 & -6 \\
 & & -4 & -2 & 0 \\
\hline
 & 2 & 1 & -3 & 0 \\
\end{array}
\]

\[ x^2 + x - 3 \]
\[ (2x + 3)(x - 1) \]
\[ x = -\frac{3}{2}, x = 1 \]

11. Use a graphing utility to approximate the real zero of the function
\[ f(x) = x^2 + 5x - 2 \] in the interval \([0, 1]\).

a. 0.259  b. 0.347  c. 0.388  d. 0.396

\[ \boxed{c. 0.388} \]
12. Use the Intermediate Value Theorem to estimate the real zero of \( f(x) = 3x^3 - 2x^2 - 2 \) in the interval \([1, 2]\).
   a. between 1.0 and 1.1
d. between 1.7 and 1.8
   b. between 1.1 and 1.2
e. none of these
c. between 1.3 and 1.4

13. Perform the indicated operation and write the results in standard form: \((5 + 4i) + (-7 - 14i) - (-12 - i)\)
   \[5 + 7 + 12 + 4i - 14i + i \]
   \[= 10 - 9i\]

14. Perform the indicated operation and write the results in standard form: \(\frac{(2-i)(3+4i)}{2+i}\)
   \[= \frac{6 + 8i - 3i - 4i^2 + i}{2 + i}\]
   \[= \frac{10 + 5i}{2 + i}\]
   \[= \frac{20 - 10i + 10i - 5i^2}{4 - 2i + 2i - i^2}\]
   \[= \frac{20 + 5}{4 + 1}\]
   \[= \frac{25}{5}\]
   \[= 5\]

15. Perform the indicated operation and write the results in standard form: \((3 - \sqrt{9})(7 + \sqrt{-4})\)
   \[= (3 - 3i)(7 + 2i) = 21 + 6i - 21i - 6i^2\]
   \[= 27 + 4i - 15i\]
   \[= 27 - 11i\]
   \[= \sqrt{27 + 4i - 15i^2}\]
   \[= \sqrt{27 + 4 - 15}\]
   \[= \sqrt{16}\]
   \[= 4\]
   \[= x^4 + 5x^2 - 36\]

16. Find a fourth degree polynomial with real coefficients that has zeros: 2, -2, 3i, -3i
   \[=(x - 2)(x + 2)(x - 3i)(x + 3i)\]
   \[=(x^2 + 4)(x^2 + 9)\]
   \[= x^4 + 13x^2 - 36\]

17. Find all the zeros of the function (real or complex). \(f(x) = x^3 + 6x^2 + 12x + 7\)
   Try \(-1\) \(\frac{1}{1} \frac{6}{-5} \frac{12}{-7} \frac{7}{10}\)
   \[= \frac{-5 \pm \sqrt{25 - 28}}{2}\]
   \[= \frac{-5 \pm \sqrt{-3}}{2}\]

   \[a = 1 \quad b = 5 \quad c = 7\]
18. Write the polynomial in **completely factored form**:
   \[ f(x) = x^4 - x^2 - 20 \]
   \[ (x^2 - 5)(x^2 + 4) \]
   \[ (x - \sqrt{5})(x + \sqrt{5})(x - 2i)(x + 2i) \]

19. Find the vertical asymptotes: \( f(x) = \frac{8x}{(x+5)^2} \)
   \[ (x + 5)^2 = 0 \]
   \[ x + 5 = 0 \quad x = -5 \]

20. Find the horizontal asymptotes (if any): \( f(x) = \frac{12x}{2-x} \)
   \[ y = \frac{12}{-1} = -12 \]

21. Match the graph with the correct function:
   a. \( f(x) = \frac{1}{x^2 - 4} \)
   b. \( f(x) = \frac{x^2}{x^2 - 4} \)
   c. \( f(x) = \frac{x}{x^2 + 4} \)
   d. \( f(x) = \frac{x}{x^2 - 4} \)

22. Use a graphing utility to help sketch the graph of: \( f(x) = \frac{x}{x^2 - 1} \) (2 points)
   Indicate all asymptote lines.
   \[ \text{vertical asymptote } x = 1 \]
   \[ \text{horizontal asymptote } y = 0 \]
   \[ x = -1 \]