1. (9 points) Explain how you could **mentally calculate** the following. Show your thinking. State the answer.

   a. \( \underline{7 + 11 + 5 + 3 + 9 + 16 + 4 + 3} = \underline{58} \)

   \[ \underline{10} \]

   b. \( 25 \times 16 = \underline{50 \times 8} = 100 \times 4 = \underline{400} \)

   c. \( 152 \div 8 = \frac{(160 - 8)}{8} = 160 \div 8 = 20 \)

   \[ - 8 \div 8 = -1 \]

   \[ \underline{19} \]

2. (6 points) Estimate the solution for each of the following computations. Show your thinking. Determine if your estimation is **more** or **less** than the actual answer and why (without actually calculating answer).

   a. \( 681 + 241 = \)

   \[ \underline{\begin{array}{c}
   680 \\
   + 240 \\
   \hline
   920
   \end{array}} \]

   \[ \frac{650}{900} \text{ too small} \]

   b. \( 284 \times 31 = \underline{300 \times 30 = 9000} \)

   \[ \frac{\text{too big}}{\text{16 too many 31's}} \]

   \[ \frac{\text{one less } 284}{\underline{8804}} \]

   \[ \text{so still too big} \]
Fill in the table above by either giving the appropriate region for each number or giving a number that fits in the given region. (10 points)

4. Label each statement as true (T) or false (F). If the statement is false, change it so that it is true (Do not change the verb). (12 points)

- T F  a. If 7 is not a factor of n, then 14 is not a factor of n.
- T F  b. If LCM(a, b) = b, then b is a multiple of a.
- T F  c. This equation is possible: \(2^3 \cdot 3^2 \cdot 17^4 = 2^3 \cdot 3^4 \cdot m\), for some whole number m. (If false, change the exponents so it is true).
- T F  d. If the sum of two numbers is odd and their product is even, then both numbers must be odd. Odd & even ≠
- T F  e. A divisibility rule for the number 12 is to see if the number is divisible by 2 and 6. 3 & 4
- T F  e. \(2^{11} \times 17^2 \times 67\) is a multiple of \(2^{11} \times 67\)
5. Let \( m = 2^3 \cdot 7^2 \), \( n = 2 \cdot 5^3 \cdot 7 \), and \( p = 3^2 \cdot 5 \cdot 7 \). Answer the following questions. (10 points)
   a. How many total factors does \( n \) have? 16 factors
   b. How many prime factors does \( p \) have? 3 prime factors
   c. List four composite factors of \( m \). 4, 8, 14, 28, 49, 56 etc
   d. What is the GCF\((m, n, p)\)? 7
   e. What is the LCM\((m, n, p)\)? \( 2^3 \cdot 3^2 \cdot 5^3 \cdot 7^2 \)

6. (4 points) Give the divisibility rule to check if a number is divisible by . . .
   a. 40
      check 5 9 8
   b. 45
      check 5 9 9

7. (6 points) List all the numbers you have to check for divisibility to see if 251 is prime. Show all work to test if it is prime. Is it prime? YES NO
   check 2, 3, 5, 7, 11, 13
   yes prime

8. Determine all possible single digits to fill in the blanks to create a number which makes each of the following true: (8 points)
   a. 3 is a factor of 4187__432
      1, 4, 7
   b. 4 is a factor of 385039__18
      none
   c. 8 is a factor of 1948572__0
      0, 4, 8
   d. 15 is a factor of 9632178__
      0
9. (5 points) Jogger A can run laps at the rate of 75 seconds per lap. Jogger B can run laps on the same track at the rate of 1 minute 30 seconds per lap. If they start at the same place and time, and run in the same direction, how long (in minutes) will it be before they are at the starting place again, at the same time?

\[
\text{LCM}(75, 90) = \text{LCM}(3 \cdot 5^2, 2 \cdot 3^2 \cdot 5) = 2 \cdot 3^2 \cdot 5^2 = 450 \text{ seconds}
\]

\[
450 \div 60 = 7 \frac{1}{2} \text{ minutes}
\]

10. (5 points) Your organization is having a fundraiser selling T-shirts at the Little League ball field. Last weekend the group had $620 in sales, and the week before they had $385 in sales. You are in charge of sales this weekend and forgot to ask how much the T-shirts are selling for. You need to figure it out and only have these numbers. How can you do that? How much do the T-shirts sell for? Show all work.

\[
\text{GCF}(237, 385) = 1 \quad \frac{11}{79} \quad \frac{272}{5 \cdot 7 \cdot 11} \quad \frac{3 \cdot 238}{6 \cdot 79} \quad \frac{237}{5 \cdot 3 \cdot 85}
\]

\$1 \text{ per shirt}

Bonus: (4 pts) Let \( a = 2^5 \cdot 3^2 \cdot 7^6 \cdot 13^4 \) and \( b = 2^2 \cdot 7^3 \cdot 13^5 \cdot 17 \). Leave answers in factored form.

a. Find the GCF\((a, b)\) and the LCM\((a, b)\).

\[
\text{GCF}\left(2^5 \cdot 3^2 \cdot 7^6 \cdot 13^4, 2^2 \cdot 7^3 \cdot 13^5 \cdot 17\right) = 2 \cdot 7 \cdot 13 \cdot 17
\]

\[
\text{LCM}\left(2^5 \cdot 3^2 \cdot 7^6 \cdot 13^4, 2^2 \cdot 7^3 \cdot 13^5 \cdot 17\right) = 2^5 \cdot 3^2 \cdot 7^6 \cdot 13^6 \cdot 17
\]

b. Multiply the GCF and the LCM.

\[
2 \cdot 7 \cdot 13 \cdot 17 \times 2^5 \cdot 3^2 \cdot 7^6 \cdot 13^6 \cdot 17 = 2^7 \cdot 3^2 \cdot 7^7 \cdot 13^7 \cdot 17
\]

c. Multiply \( a \) and \( b \).

\[
2^5 \cdot 3^2 \cdot 7^6 \cdot 13^4 \cdot 2^2 \cdot 7^3 \cdot 13^5 \cdot 17 = 2^7 \cdot 3^2 \cdot 7^9 \cdot 13^{10} \cdot 17
\]

d. Use your answers from (b) and (c) to come up with a general equation relating \( a \) and \( b \) to the GCF and LCM.

\[
\text{GCF}(a, b) \times \text{LCM}(a, b) = a \times b
\]