MATH 313
EXAM 3

Follow all directions. Use complete sentences for explanations. Point values are indicated. Relax, breathe, and good luck 😊

1. (20 points) Find the next 4 terms in the patterns below. Find the given term if requested.
   a. 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89
   b. ▲, ▼, ▲, ▼, ▲, ▼, ▲, ▼, ▲, ▼
   c. □, ◻, □, ◻, □, ◻, □, ◻, □, ◻, □, ◻
   d. \(\frac{1}{3}, \frac{2}{4}, \frac{3}{5}, \frac{4}{6}, \frac{5}{7}, \frac{6}{8}, \frac{7}{9}\)
   e. Find the 86th term in sequence b = __________
   f. Find the 100th term in sequence d = \(\frac{100}{102}\)

2. (3 points) What digit is in the one’s place in the calculated form of \(7^3\)?
   The one’s place digit will be __________
   \(4 \, 135\, _{32} \, r \, 3\)

3. (4 points) Draw or give an example of a graph or mapping that is a function and a graph or mapping that is not a function.

Function:

NOT a function
4. (15 points) The first 3 terms of a figurate patterns is shown below. Look for patterns that would give you a rule to help draw the 4th stage, and find the number of square tiles in the 5th, 10th, and n\textsuperscript{th} terms in the sequence.

I) H pattern:

\[ \begin{array}{c}
\text{stage : 1} \\
\text{7}
\end{array} \quad \begin{array}{c}
\text{stage : 2} \\
\text{12}
\end{array} \quad \begin{array}{c}
\text{stage : 3} \\
\text{17}
\end{array} \quad \begin{array}{c}
\text{stage : 4} \\
\text{22}
\end{array} \]

a. Draw in the 4th stage

b. How many squares in the fifth term? \( \frac{27}{52} \)

c. How many squares in the tenth term? \( 5n + 2 \)

d. How many squares in the nth term? \( 5n + 2 \)

e. Justify your function by relating it to the figures above. Show where the "n" plays a role physically in each figure.

\[
\text{5 groups of } n \text{ plus 2 squares at intersection points (with x)}
\]

5. (6 points) Find the following sum of even numbers as a function in terms of n. Hint: making a table of the number of terms and their sum may help.

\[
2 + 4 + 6 + \ldots = n(n+1)
\]

<table>
<thead>
<tr>
<th>n</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>12</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>6</td>
<td>6</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>3</td>
<td>12</td>
<td>12</td>
<td>12</td>
<td>12</td>
</tr>
<tr>
<td>4</td>
<td>20</td>
<td>20</td>
<td>20</td>
<td>20</td>
</tr>
</tbody>
</table>
6. (20 points) Find the rules for the following functions. Hint: look for patterns including common differences and multiples.

   a. 
<table>
<thead>
<tr>
<th>x</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td></td>
<td>38</td>
<td>34</td>
<td>30</td>
<td>26</td>
<td>22</td>
</tr>
</tbody>
</table>

   The function rule is \( y = \frac{-4x + 42}{-38} \).

   This pattern is an example of a(n) arithmetic sequence.

   The 20th term in this sequence is \(-80 + 42\).

   b. 
<table>
<thead>
<tr>
<th>n</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>f(n)</td>
<td>3</td>
<td>6</td>
<td>12</td>
<td>24</td>
<td>48</td>
</tr>
</tbody>
</table>

   The function rule is \( f(n) = 3 \cdot 2^{n-1} \).

   This pattern is an example of a(n) geometric sequence.

   192 is the 7th term in the sequence.

   \[
   \frac{192}{3} = 2^{n-1} \quad \text{or} \quad \frac{64}{3} = 2^{n-1} \quad \text{or} \quad \frac{192}{18} = \frac{64}{3} = \frac{18}{12}
   \]

   c. 
<table>
<thead>
<tr>
<th>x</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>y</td>
<td></td>
<td>-6</td>
<td>-3</td>
<td>2</td>
<td>5</td>
<td>18</td>
</tr>
</tbody>
</table>

   The function rule is \( y = x^2 - 7 \).

   This pattern is an example of a(n) quadratic sequence.

   d. 
<table>
<thead>
<tr>
<th>n</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
</tr>
</thead>
<tbody>
<tr>
<td>f(n)</td>
<td></td>
<td>5</td>
<td>9</td>
<td>15</td>
<td>23</td>
<td>33</td>
</tr>
</tbody>
</table>

   The function rule is \( f(n) = n^2 + n + 3 \).

   The 10th term in the sequence is \( 100 + 10 + 3 = 113 \).
7. (5 points) Below is a series of toothpick houses built by students. Write a formula for the number of toothpicks needed to build the nth toothpick house.

House 1  |  House 2  |  House 3
---------|-----------|-----------
5        | 9         |           

The nth toothpick house will take \(4n + 1\) toothpicks to build.

8. (5 points) Using the toothpick houses in the problem above, your student Ronnie comes up to you with her rule for the number of toothpicks. She says there is one floor part for each house or \(n\) toothpicks across the bottom, two roof parts or \(2n\) toothpicks across the top and one extra side piece or \(n + 1\) toothpicks for the sides of the houses, so there must be \(n + 2n + n + 1\) toothpicks used. **Comment on her thinking**, is she mathematically correct? How does her formula compare to your formula above?

- Good thinking - very visual
- Formulas will match

9. (4 points) Find a function rule that give the perimeter of the nth shape in the following pattern:

Perimeter = \(7n + 9\)
10. (12 points) Use the function machines below to find the outputs for the machine or combination of machines in the given order, and the input.

<table>
<thead>
<tr>
<th>Machine 1</th>
<th>Machine 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Multiply the input by 2, then subtract 7</td>
<td>Square the input</td>
</tr>
</tbody>
</table>

a. Input = 5 into machine 2
Output = \( 2 \cdot 5 = 10 \)

b. Input = 5 first into machine 1, then into machine 2
Output = \( 3^2 = 9 \)

c. Input = 5 into machine 2, then into machine 1
Output = \( \frac{2(25) - 7}{1} = 43 \)

d. Input = \( x \) into machine 1
Output = \( 2x - 7 \)

e. Input = \( x \) first into machine 1, then into machine 2
Output = \( (2x - 7)^2 \)

\( 4x^2 - 28x + 49 \)

f. Input = \( x \) first into machine 2, then into machine 1
Output = \( 2x^2 - 7 \)

11. (6 points) Suppose a rectangular “hot plate” is made from square tiles placed side by side in a row (just one row). Each tile is 3 in. x 3 in. square. Find a function rule for the perimeter of this “hot plate”. Hint: a picture and table may help.

<table>
<thead>
<tr>
<th>Number</th>
<th>Perimeter</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>12</td>
</tr>
<tr>
<td>2</td>
<td>18</td>
</tr>
<tr>
<td>3</td>
<td>24</td>
</tr>
<tr>
<td>4</td>
<td>30</td>
</tr>
</tbody>
</table>

Perimeter = \( 6n + 16 \)