Functions

A function is a set of ordered pairs in which no x-coordinate is repeated. There are various ways to identify functions, depending on the type of information you are given.

1. Given a set of ordered pairs, check the x-coordinates. If no x-coordinate is repeated, the set of ordered pairs describes a function.
2. Given a graph, draw vertical lines through the graph. If any vertical line intersects more than 1 point on the graph, the graph does not describe a function. (This is called the vertical line test.)
3. Given a mapping diagram, each element in the first set must map to only one element in the second set.
4. Given an equation, graph the equation and use the vertical line test to determine whether the equation represents a function.

Identify which of the following are functions.

1. \{(2, 5), (1, 3), (2, -5)\}
2. \{(0, 1), (2, 1), (3, 4)\}
3. \[
\begin{array}{ccc}
-5 & -4 & -3 \\
-2 & -1 & 1 \\
-3 & -4 & -5
\end{array}
\]

6. \[
\begin{array}{ccc}
0 & 1 & 2 \\
1 & 2 & 3
\end{array}
\]

Domain \quad Range

7. \[
\begin{array}{ccc}
0 & 1 & 2 \\
1 & 2 & 3
\end{array}
\]

Domain \quad Range

8. \[
\begin{array}{ccc}
0 & 1 & 2 \\
1 & 2 & 3
\end{array}
\]

Domain \quad Range
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Finding the Domain of a Function

The domain of a function is the set of all permissible \( x \)-values.
Given a set of ordered pairs, the domain is the set of \( x \)-coordinates.
Given an equation:
1. Often the domain is the set of all real numbers.
2. If there is a variable in the denominator, set the denominator equal to 0 and solve. The domain consists of all real numbers except the value(s) that makes the denominator 0.
3. If there is a variable in a square root (or an even indexed root), set the radicand greater than or equal to 0 and use that solution as the domain.

Find the domain.

11. \( \{(2, 4), (1, 3), (3, 5)\} \)
12. \( y = 4x - 1 \)
13. \( y = \frac{6}{3x - 1} \)
14. \( y = \sqrt{x + 2} \)
15. \( y = \frac{\sqrt{x + 4}}{x} \)
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### Function Notation

Function notation consists of a function name, usually $f$, $g$, or $h$, followed in parentheses by the variable that represents the domain elements: $f(x)$, $g(x)$, $h(x)$ read $f$ of $x$, $g$ of $x$ and $h$ of $x$.

Note that $f(x)$ does not mean $f$ times $x$.

Finding $f(3)$ means finding the $y$-value paired with the $x$-value 3.

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For example, given $f(x) = 2x + 1$, then $f(3)$ is obtained by replacing $x$ with 3 in $2x + 1$.

16. Find $f(3)$ if $f(x) = 2x + 1$

17. Find $g(-2)$ if $g(x) = x^2 - 2x + 1$

18. Find $f(a)$ if $f(x) = 4x - 6$

19. Find $f(a + h)$ if $f(x) = x^2 + 4x$
Examples.

Given \( f(x) = 3x - 7 \), find \( f(-2) \).
\[
f(-2) = 3(-2) - 7 = -6 - 7 = -13
\]

Given \( f(x) = 2x^2 - 5x + 2 \), find \( f(0) \).
\[
f(0) = 2(0)^2 - 5(0) + 2 = 2 - 0 - 0 + 2 = 2
\]

Find the indicated function values.

1. \( f(x) = 2x + 5 \)
   a) \( f(-2) = \) __________
   b) \( f(-8) = \) __________
   c) \( f(0) = \) __________
   d) \( f(1.2) = \) __________
   e) \( f\left(\frac{3}{4}\right) = \) __________

2. \( g(t) = t^2 - 5 \)
   a) \( g(0) = \) __________
   b) \( g(7) = \) __________
   c) \( g(-9) = \) __________
   d) \( g(-1.4) = \) __________
   e) \( g\left(\frac{2}{3}\right) = \) __________

3. \( h(x) = -22 \)
   a) \( h(-11) = \) __________
   b) \( h(-1.6) = \) __________
   c) \( h(0) = \) __________
   d) \( h(15) = \) __________
   e) \( h(209) = \) __________

4. \( f(x) = |x| - 8 \)
   a) \( f(-19) = \) __________
   b) \( f(-1) = \) __________
   c) \( f(0) = \) __________
   d) \( f(18) = \) __________
   e) \( f(100) = \) __________

5. \( g(t) = |t - 2| \)
   a) \( g(7) = \) __________
   b) \( g(-5) = \) __________
   c) \( g(-30) = \) __________
   d) \( g(400) = \) __________
   e) \( g(a+1) = \) __________

6. \( f(x) = 2x^3 - x \)
   a) \( f(0) = \) __________
   b) \( f(4) = \) __________
   c) \( f(-3) = \) __________
   d) \( f(4a) = \) __________
   e) \( f(-10) = \) __________
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Graphing Functions
To graph a function, prepare a table of values. The function $f(x)$ can be thought of as $y$. Although you need only two points to graph a line, to graph other functions, you will probably need more than two points. Functions may be identified by names:

- Linear function: $f(x) = ax + b$, graph is a nonhorizontal line if $a \neq 0$
- Constant function: $f(x) = c$, graph is a horizontal line
- Quadratic function: $f(x) = ax^2 + bx + c$, $a \neq 0$, graph is a parabola
- Polynomial function: $f(x)$ = polynomial in $x$, graph is a smooth curve that may contain peaks and valleys
- Square Root function: $f(x) = \sqrt{ax + b}$, $a \neq 0$, graph is one branch of a parabola

identify the type of function.

29. $f(x) = \frac{1}{2}x - 3$
30. $f(x) = x^2 + 2$
31. $f(x) = -4$
32. $f(x) = \sqrt{x} + 2$
33. $f(x) = x^3 - 1$
34. $f(x) = -\frac{3}{2}x + 3$
Graph each function. **state the range.**

7. \( f(x) = 4x + 2 \)

8. \( f(x) = -3x - 1 \)

9. \( g(x) = \frac{2}{3}x + 4 \)

10. \( f(x) = \frac{1}{4}x - 2 \)

11. \( h(x) = -|2x| \)

12. \( f(x) = x^3 \)

Determine whether the graph is that of a function. **If a function state domain & range.**

13. 

14. 

15. 
Operations with Functions

We add, subtract, multiply and divide functions using the following definitions.

\[(f + g) (x) = f(x) + g(x)\]
\[(f - g) (x) = f(x) - g(x)\]
\[(f \cdot g)(x) = f(x) \cdot g(x)\]
\[\left(\frac{f}{g}\right)(x) = \frac{f(x)}{g(x)}, g(x) \neq 0\]

If \(f(x) = 3x - 6\) and \(g(x) = 2x^2 + x - 1\), find:

20. \((f + g)(2)\)
21. \((f - g)(-4)\)
22. \((f \cdot g)(x)\)
23. \(\left(\frac{f}{g}\right)(1)\)