Multiplication properties of exponents

When multiplying exponents, it is important to remember the following properties:

1. When multiplying powers having the same base, add the exponents, keeping the same base. (Remember: a^3, a is the base, 3 is the exponent, and a^3 is the power.)
   For example, x^3 \cdot x^5 = x^{3+5} = x^8.

2. When finding a power of a power, multiply the exponents. For example, (x^3)^2 = x^6.

3. When finding the power of a product, find the power of each factor and multiply.
   For example, (x \cdot y)^2 = x^2 \cdot y^2.

Simplify (5x^2)(xy)^3
   = (5x^2)(x^3y^3)
   = 5x^5y^3

Simplify (2x^4)(-x^2)^3
   = (2x^4)(-x^6)
   = -8x^{10}

1. What do you do with the exponents when multiplying powers that have the same base?

2. Label the base, the exponent, and the power in x^3.

3. Explain what you are to do when finding the power of a product.

Simplify each expression.

4. 3x \cdot x^2

5. (-6xy)^2(x^2y)^3

6. (4z^4)^2(2x^2y)(-3xy^3z^5)

7. -4x^4 \cdot x^3

8. (4x^3y^2)^3(-2x^2y^4)

9. (3x)^2(2x^3y^5)(-5x^5y^2)

10. (-x^2)(-x)^2

11. -xy(-xy)^2

12. (3x^3)(5x^5)

13. (-2x^3y^2z)^4(2xyz^4)^2
Balloon Baffled

The largest latex balloon to ever appear in the Macy’s Thanksgiving Day parade in New York City was 61 feet long and 35 feet wide. It held 18,907 cubic feet of helium and air. Who did this balloon depict?

To find out, simplify each expression. Write the problem number in front of the corresponding answer listed in the table. To spell out the answer at the bottom of the page, refer to the table and write the code letter that corresponds to the problem number given.

1. \((3x)(x^4)\)
2. \((4x^2)(8x^3)\)
3. \((x^5)^3\)
4. \((3x)(2x^2)(5x^3)\)
5. \((-3x^2)(-7x)(-2x^4)\)
6. \((-x^2y)(-xy^2)(-xy^4)\)
7. \((-12x)(3x^2y)(-2y)\)
8. \((-x^2)^3(4x^2)^3\)
9. \((-3x)^3(x^2y)^4\)
10. \((xy)^2(x^3)^4\)
11. \((2x^4y)^3\)

<table>
<thead>
<tr>
<th>Code Letter</th>
<th>Problem #</th>
<th>Answer</th>
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<tr>
<td>A</td>
<td>30x^6</td>
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<tr>
<td>C</td>
<td>72x^3y^2</td>
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<tr>
<td>D</td>
<td>x^14y^2</td>
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<tr>
<td>E</td>
<td>32x^5</td>
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<td>F</td>
<td>-x^4y^7</td>
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<tr>
<td>G</td>
<td>3x^5</td>
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</tr>
<tr>
<td>H</td>
<td>-64x^12</td>
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</tr>
<tr>
<td>I</td>
<td>x^15</td>
<td></td>
</tr>
<tr>
<td>L</td>
<td>8x^12y^3</td>
<td></td>
</tr>
<tr>
<td>R</td>
<td>-42x^7</td>
<td></td>
</tr>
<tr>
<td>T</td>
<td>-3x^3y^7</td>
<td></td>
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</table>

58 Multiplying exponents
**Simplifying polynomials**

A monomial is an expression that can be a constant, a variable, or a product of a constant and one or more variables. A polynomial is a monomial or the sum of monomials. Specifically, a binomial is the sum of two monomials, and a trinomial is the sum of three monomials. Remember, like terms are terms that have the same variable(s) raised to the same power(s). A polynomial is considered to be in simplest form when there are no like terms. The degree of a term is the actual power of that term. A constant has a degree of 0. The degree of a polynomial is the largest degree of its terms.

1. Give the degree of the polynomial $3x^4 + 2x^2 - 1$.
   - The degree is 4.
   - The largest degree of all three terms is 4.

2. Simplify $3x^3 + 4x - 2x^2 - 5x + 10$ and give its degree.
   - $x^3 - x + 10$
   - The degree is 5.
   - Add like terms and put in simplest form.
   - The greatest degree of all three terms is 5.

3. Simplify $4x^8 + 7x^4 - 8x^3 + 3x^3 - 9x^6 + x^4$ and put answer in standard form.
   - $-5x^6 + 8x^4 - 5x^3$
   - $8x^4 - 5x^6 - 5x^3$
   - Add like terms and put in simplest form.
   - Put terms in descending order; from largest degree to the smallest degree, which is standard form.

Give the definition of each term and give an example of each.

1. monomial
2. trinomial
3. standard form
4. binomial
5. degree
6. polynomial

Simplify each polynomial. Give the degree of the polynomial. Make sure each answer is in standard form.

7. $6x^3 - 4x^2 + 2x - 7x^3 + 5x$
8. $x^5 + 4x^2 - 7x^3 + 6x^2 - 2x^6 + 3x^3$

9. $10 - 7x^6 + 2x^4 - x^4 + 8x^5 - 3x^3$
10. $10x + 3x^2 - 8x + 6x^2$

11. $10x^4 - 7x^3 + 5x^3 - 4x^4 - 8x^4$
12. $-x^3 + 4x^2 - 7 + 3x^2 - 8x^3$

13. $4x^2 - 3x^5 + x^7 + 5x^6 + 4x^4 - 6$
14. $-6x + 5x^3 - 4x^2 + 3 + 9x^3 - 10x + 6x - 5$
Adding and subtracting polynomials

When adding polynomials, the following two methods can be used.

1. The horizontal method:
   Add \((3x^4 + 8x^2 - 2)\) and \((-4x^3 + 7x^2 + 5)\)
   First group like terms in descending order: \((3x^4 + 7x^2) + (8x^2 - 4x^3) + (-2 + 5)\)
   Simplify. \(10x^4 + 4x^2 + 3\)

2. The vertical method:
   Add \((4x^7 + 6x^4 - 10 + 2x^6)\) and \((10x^5 - 5x^7 - 3x^4 + 12)\)
   Group like terms in columns in descending order:
   \[
   \begin{align*}
   &4x^7 + 2x^6 + 6x^4 - 10 \\
   &+ 5x^7 + 10x^5 - 3x^4 + 12 \\
   &= -x^7 + 12x^5 + 3x^4 + 2
   \end{align*}
   \]
   Simplify.

When subtracting polynomials, change the sign of each term in the second polynomial and then choose from one of the methods above to simplify.

Subtract. \((10x^7 - 4x^4 + 7x^2 + 2) - (12x^4 + 6x^7 + 8 - 5x^2)\)
Change sign of each term in second polynomial. \(10x^7 - 4x^4 + 7x^2 + 2 - 12x^4 - 6x^7 - 8 + 5x^2\)
Choosing method 1 above, group like terms in descending order:
\((10x^7 - 6x^7) + (-4x^4 - 12x^4) + (7x^2 + 5x^2) + (2 - 8)\)
Simplify. \(4x^7 - 16x^4 + 12x^2 - 6\)

1. What is important to remember to do when subtracting polynomials?

Simplify each problem by adding and subtracting.

2. \((7x - 5) - (3x + 7)\)
3. \((2x^2 - 3x + 1) - (5x^2 - 3x + 10)\)
4. \((6x + 7) + (8x - 3)\)
5. \((x^3 - 7x^4 + 2x^2 - 3x) + (4x^4 - 8x^3 + 5x)\)
6. \((9x^2 + 3x - 5) - (3x^2 + 4x - 10)\)
7. \((8x^4 - 9x^3 + 2x) - (6x^4 + 7x^3 - 3x)\)
8. \((6x^2 + 9x + 4) - (-7x^2 + 2x - 1)\)
9. \((x^3 - x^2 + 2) - (x^3 + x^2 + 5)\)
10. \((-8x^2 + 2) + (7x - 5)\)
11. \((8x + 6) - (10x + 16) + (4x - 6)\)
12. \((6x^2 + 7x - 9) + (3x + 8)\)
13. \((4x^2 + 2) + (3x^2 - 4x + 6) - (5x^2 + 10)\)
**Goofin’ With Golf**

Why did the golfer need a new club?

To find out, add the polynomials below. Write the problem number in front of the corresponding answer listed in the table. To spell out the words at the bottom of the page, refer to the table and write the code letter that corresponds to the problem number given.

1. \((2x^2 - 7x + 6) + (-3x^2 + 7x)\)
2. \((x^2 + 2x) + (2x^2 - 3x + 4)\)
3. \((3x^2 + 2x - 4) + (-x^2 + 2x - 3)\)
4. \((4x^2 - 5x + 4) + (-4x^2 + 5x - 4)\)
5. \((5x^2 + 3x - 5) + (2x^2 - 5x + 7)\)
6. \((3x^2 + 2x - 7) + (-2x^2 + 15)\)
7. \((x^2 + 5x + 13) + (-3x^2 + 2x - 8)\)
8. \((3 + 2x + x^2) + (5 - 8x + x^2)\)
9. \((5x^2 - 4) + (3x^2 + 8x + 4)\)
10. \((-7 - x + 7x^2) + (11 + x^2)\)
11. \((4x^2 - 7x - 2) + (2x^2 - 9x + 4)\)
12. \((7 - 5x + 6x^2) + (11 - x - 5x^2)\)

<table>
<thead>
<tr>
<th>Code Letter</th>
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<th>Answer</th>
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<tbody>
<tr>
<td>A</td>
<td>8 - 6x + 2x^2</td>
<td></td>
</tr>
<tr>
<td>B</td>
<td>2x^2 + 4x - 7</td>
<td></td>
</tr>
<tr>
<td>C</td>
<td>6x^2 - 16x + 2</td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>-x^2 + 6</td>
<td></td>
</tr>
<tr>
<td>E</td>
<td>x^2 + 2x + 8</td>
<td></td>
</tr>
<tr>
<td>H</td>
<td>18 - 6x + x^2</td>
<td></td>
</tr>
<tr>
<td>I</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>L</td>
<td>3x^2 - x + 4</td>
<td></td>
</tr>
<tr>
<td>N</td>
<td>-2x^2 + 7x + 5</td>
<td></td>
</tr>
<tr>
<td>O</td>
<td>4 - x + 8x^2</td>
<td></td>
</tr>
<tr>
<td>S</td>
<td>7x^2 - 2x + 2</td>
<td></td>
</tr>
<tr>
<td>U</td>
<td>8x^2 + 8x</td>
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Adding polynomials

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Presidential Power

In 1961, at the age of 43 years and 236 days, this President became the youngest elected President in United States history. What is the name of this famous man?

To find out, solve each problem. From the list of answers provided, circle the letters that are next to the matching solutions. Write the letters in front of their corresponding problem numbers to spell out the answer.

1. \((5x^2 - 3x + 6) - (2x^2 - 3x - 2)\)
2. \((-3x^2 - 6x + 2) - (-4x^2 - x - 7)\)
3. \((6x^2 - 7x + 1) - (5x^2 - 3x - 2)\)
4. \((5x^2 + 7x + 2) - (-5x^2 + 7x - 3)\)
5. \((2x^2 - 6x + 5) - (5x^2 - 6x - 3)\)
6. \((5x^2 + 4) - (x^2 - 3x + 2)\)
7. \((3x^2 - 7x + 2) - (x^2 + 8x + 5)\)
8. \((3x^2 - 8) - (5x^2 + 2x + 7)\)
9. \((6x^2 - 3x - 7) - (5x^2 + 2x + 3)\)
10. \((7x^2 - x - 7) - (x^2 + 11)\)
11. \((5x^2 + 3x) - (2x^2 - 8x + 4)\)
12. \((4x + 1) - (x^2 - 2x + 3)\)

Circle the answers:

N. -2x^2 - 2x - 15
O. x^2 - 5x + 9
D. 3x^2 + 11x - 4
F. -3x^2 + 8
A. 3x^2 - 1
J. 3x^2 + 8
Y. -x^2 + 6x - 2
B. -x^2 - 7x - 7
N. 10x^2 + 5
E. 2x^2 - 15x - 3
H. x^2 - 4x + 3
L. x - 5
G. x^2 + 10
K. 4x^2 + 3x + 2
N. x^2 - 5x - 10
E. 6x^2 - x - 18

Answer: 

Subtracting polynomials
Evaluate each expression in a row using the given value for H. Circle the letter of the expression with the “highest” (greatest) value. Write the letter above the problem number in the Fun Fact.

1. Let H = 2
   R. $-H^2 = ____$
   S. $H^3 = ____$
   T. $-H^3 = ____$

2. Let H = 0.2
   D. $H^2 = ____$
   E. $5H = ____$
   F. $5H^2 = ____$

3. Let H = -3
   Y. $-H^2 = ____$
   M. $H^3 = ____$
   W. $-H^3 = ____$

4. Let H = -1
   V. $1 - 2H = ____$
   U. $1 - H^2 = ____$
   R. $1 - H^3 = ____$

5. Let H = $\frac{1}{2}$
   A. $4H = ____$
   B. $4H^2 = ____$
   C. $4H^3 = ____$

Fun Fact: One hundred twelve feet is the height of the highest officially recorded

___ ______ ______ ______ ______

Evaluate each expression in a row using the given value for L. Circle the letter of the expression with the “lowest” (least) value. Write the letter above the problem number in the Fun Fact.

6. Let L = 2
   J. $3^L = ____$
   E. $-3^L = ____$
   Y. $-L^3 = ____$

7. Let L = -2
   M. $-L^2 = ____$
   O. $L^3 = ____$
   R. $-L^3 = ____$

8. Let L = $\frac{1}{2}$
   K. $L^2 = ____$
   A. $-L^2 = ____$
   T. $(1 - L)^2 = ____$

9. Let L = 0.2
   B. $-L^2 = ____$
   C. $-5L^2 = ____$
   D. $5L^2 = ____$

10. Let L = -1
    T. $L^2 = ____$
    R. $(-L)^2 = ____$
    N. $-L^2 = ____$

Fun Fact: In 1954 humans first reached the floor of the

( depth of 13,827 feet).
Multiplying polynomials by a monomial

To multiply a multi-termed polynomial by a monomial, simply multiply each term in the polynomial by the monomial using the distributive property. Remember the properties of exponents when simplifying such problems.

Multiply

1. $3x(3x^3 - 2x^2 + 8x - 7)$
2. $3x(3x^2) - 3x(2x^2) + 3x(8x) - 3x(7)$
3. $9x^4 - 6x^3 + 24x^2 - 21x$

Distribute $3x$ to each term.
Multiply to simplify.

You may have to simplify an expression using the distributive property to multiply the polynomial by a monomial.

Simplify

1. $4x^7 + 9x^4 - 7 + 2x^2(8x^3 - 5x^2)$
2. $4x^7 + 9x^4 - 7 + 2x^2(8x^3) - 2x^2(5x^2)$
3. $4x^7 + 9x^4 - 7 + 16x^3 - 10x^4$
4. $20x^7 - x^4 - 7$

Distribute the $2x$.
Multiply to begin simplifying.
Simplify.

Multiply each expression.

1. $6x(4x - 3)$
2. $4x(x^2 - 6x + 3)$
3. $(-3x^2 + 4x - 2)(-4x^3)$
4. $x(-5x + 2)$
5. $-3x(5x^2 - 2x - 6)$
6. $(-x^2 + 2x - 1)(5x^3)$
7. $-2x(-4x - 8)$
8. $-x^2(7x^2 - x + 4)$
9. $-5x^4(2x^2 - 8x + 6)$

Simplify each expression.

10. $-4x^2 - 5x + 7 + 3(x^2 + 8x - 2)$
11. $6x^2 + 3(x - 5) - 8x$
12. $5x^2 - 3x(3x - 7)$
13. $2x^2 + 7x + 6 - 5x(-2x - 1)$
14. $6x^2 + 4x + (7x - 3)2x$
15. $-3x^2 - 3x(4x^2 - 5x + 7) - 8x^2$
16. $-x^2 + 8x - 6 - 7(3x^2 - 5x + 9)$
17. $x^3 + (7x^2 - 9x - 1)x + 10x^2$
The product $15 \times 14$ can be shown geometrically by finding the area of each region.

1. So $15 \times 14 = \underline{\quad} (100) + \underline{\quad} (10) + \underline{\quad} (1)$ which equals \underline{\quad}.

2. Complete the figure to show the product $(x + 2)(x + 3)$.

3. So $(x + 2)(x + 3) = \underline{\quad} (x^2) + \underline{\quad} (x) + \underline{\quad} (1)$ which equals \underline{\quad}.

4. Use the information given to fill in the blanks.

5. Find the area of the shaded region. \underline{\quad}
The product \((a + b)^2\) can be pictured as a square with sides \(a + b\) as in the figure at right:

The area of the large square is \((a + b)^2\). It is also the sum of the areas of the four smaller squares. Hence, \((a + b)^2 = a^2 + ab + ab + b^2 = a^2 + 2ab + b^2\).

Similarly, the product \((x + 3)(x + 2)\) could be pictured as a rectangle:

\[
\begin{array}{c|c}
  x & 3 \\
  \hline
  x^2 & 3x \\
  2x & 6 \\
\end{array}
\]

We see that \((x + 3)(x + 2) = x^2 + 5x + 6\).

**Label the rectangles below to represent the given product. Then state the trinomial that is the product.**

1. \((x + 5)(x + 4)\)

2. \((2x + y)(x + 3y)\)

3. \((4x + 7)(x + 6)\)

4. \((3x + 4)(2x + 5)\)

5. \((5x + 3)(3x + 4)\)

6. \((x - 4)(x + 7)\)

7. Explain what's wrong with using this geometric representation of area for problem 6.
Multiplying binomials

The easiest way to multiply binomials is to use what is called the FOIL method. This method multiplies the first terms of the binomials (F), multiplies the outer terms of the binomials (O), multiplies the inner terms of the binomials (I), multiplies the last terms of the binomials (L), and simplifies by adding like terms.

1. Multiply $(3x - 2)(4x + 3)$
   
   \[
   \begin{align*}
   3x(4x) &= 12x^2 & \text{Multiply first terms.} & \quad F \\
   3x(3) &= 9x & \text{Multiply outer terms.} & \quad O \\
   -2(4x) &= -8x & \text{Multiply inner terms.} & \quad I \\
   -2(3) &= -6 & \text{Multiply last terms.} & \quad L \\
   12x^2 + 9x - 8x - 6 &= \text{Simplify by combining like terms.} \\
   12x^2 + x - 6 &= \text{Final product}
   \end{align*}
   \]

2. Multiply $(5x + 2)(7x - 3)$
   
   \[
   \begin{align*}
   5x(7x) + 5x(-3) + 2(7x) + 2(-3) &= \text{Multiply.} \\
   35x^2 - 15x + 14x - 6 &= \text{Final product}
   \end{align*}
   \]

Multiply.

1. $(x + 7)(x - 5)$
2. $(x - 5)(x - 5)$
3. $(9x - 1)(6x + 2)$
4. $(7x - 8)(8x - 7)$
5. $(x + 2)(x + 3)$
6. $(x + 7)(x - 7)$
7. $(3x + 5)(-4x - 7)$
8. $(-3x - 9)(-x - 6)$
9. $(x - 10)(x + 1)$
10. $(2x - 3)(5x + 4)$
11. $(-5x + 6)(x - 2)$
12. $(-4x + 4)(5x + 8)$

Simplify each expression.

13. $(3x^2 - 4y)(2x^2 + 5y)$
14. $(a^2 + b^2)(x^2 + y^2)$
15. $(4x^3 + 6x^2)(8x^3 - x^2)$
16. $(7x - 2)(x^2 + 3)$

17. $(a + b)(x + y)$
18. $(x^2 - 8)(x^2 + 5)$
Multiplying polynomials in two special cases

When multiplying polynomials, it is important to remember two special products.

1. Sum and difference product: \((a + b)(a - b) = a^2 - b^2\)
   - Multiply \((5x + 2)(5x - 2)\)
   - \(a = 5x, \ b = 2\)
   - \((5x)^2 - 2^2\)
   - Square both terms.
   - The result is special product.

2. Square of a binomial product: \((a + b)^2 = a^2 + 2ab + b^2\)\((a - b)^2 = a^2 - 2ab + b^2\)
   - Multiply \((3x + 2)^2\)
   - \(a = 3x, \ b = 2\)
   - \((3x)^2 + 2(3x)(2) + 2^2\)
   - Square \(a\), square \(b\), and multiply 2 times \(a\) and \(b\).
   - The result is special product.
   - Multiply \((4x - 5)^2\)
   - \(a = 4x, \ b = 5\)
   - \((4x)^2 - 2(4x)(5) + 5^2\)
   - Square \(a\), square \(b\), and multiply 2 times \(a\) and \(b\).
   - The result is special product.

Multiply each expression.

1. \((x - 8)^2\)
2. \((2x + 4)^2\)
3. \((x^2 + 3)^2\)

4. \((3x - 2)(3x + 2)\)
5. \((x - 6)^2\)
6. \((x^2 - 4)^2\)

7. \((x + 7)(x - 7)\)
8. \((7x - y)(7x + y)\)
9. \((4x + 1)(4x - 1)\)

10. \((x - 9)(x + 9)\)
11. \((8x - 2y)^2\)
12. \((2 + 6x)(2 - 6x)\)

Solve each equation for \(x\).

13. \((x - 7)(x + 2) = (x - 4)^2\)
14. \((x - 4)^2 = (x + 2)^2\)
15. \((x - 8)^2 = x^2\)

16. \((x + 5)^2 = (x + 5)(x - 5)\)
17. \(x^2 = (x + 10)^2\)
18. \((x + 6)(x - 6) = (x + 6)^2\)
To find out, simplify each expression by multiplying. Match each problem to its solution. Write the letter corresponding to its solution above each problem number at the bottom of the page to spell out the answer.

1. \((x + 1)(2x + 2)\)
2. \((x + 3)(x - 2)\)
3. \((4x + 7)(3x - 8)\)
4. \((x - 3)(x + 4)\)
5. \((3x + 11)(5x - 2)\)
6. \((2x + 3)(x^2 + 3x + 8)\)
7. \((2x + 5)(3x^2 - 8x + 7)\)
8. \((3x - 5)(5x + 2)\)
9. \((13x - 3)(13x + 3)\)
10. \((3x + 9)^2\)
11. \((4x - 2)(4x + 2)\)
12. \((2x - 1)(x + 8)\)
13. \((7x + 2)(5x + 1)\)
14. \((2x^2 + 7x - 11)(3x - 5)\)
15. \((4x + 5)^2\)

L. \(15x^2 + 49x - 22\)
T. \(9x^2 + 54x + 81\)
I. \(2x^2 + 15x - 8\)
O. \(x^2 + x - 6\)
P. \(6x^3 - x^2 - 26x + 35\)
H. \(6x^3 + 11x^2 - 68x + 55\)
E. \(2x^3 + 9x^2 + 25x + 24\)
N. \(16x^2 - 4\)
U. \(12x^2 - 11x - 56\)
T. \(16x^2 + 40x + 25\)
S. \(x^2 + x - 12\)
Y. \(2x^2 + 4x + 2\)
G. \(35x^2 + 17x + 2\)
T. \(15x^2 - 19x - 10\)
A. \(169x^2 - 9\)
Why Does Batman Brush His Teeth So Often?

Use long division to divide the polynomials below. Find the answer in the answer column and notice the letter next to it. Write this letter in each box that contains the number of that problem. Keep working and you will discover the answer to the title question.

<table>
<thead>
<tr>
<th>Problem</th>
<th>Answer</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. ((6x^3 - 2x^2 - 8x) ÷ (2x))</td>
<td>O. (x^4 - 2x^3 - 3)</td>
</tr>
<tr>
<td>2. ((2x^2 - x - 15) ÷ (x - 3))</td>
<td>R. (8x^3 + 7x^2 - 18x)</td>
</tr>
<tr>
<td>3. ((3x^2 - 25x + 28) ÷ (x - 7))</td>
<td>N. (3x^2 - x - 4)</td>
</tr>
<tr>
<td>4. ((x^3 + 5x^2 + 11x + 10) ÷ (x + 2))</td>
<td>H. (3)</td>
</tr>
<tr>
<td>5. ((x^5 - 2x^4 - 3x) ÷ (x))</td>
<td>B. (2x^3 - 3x^2 + 5x - 7)</td>
</tr>
<tr>
<td>6. ((3x^3 + 14x^2 + 4x - 32) ÷ (3x + 8))</td>
<td>P. (x^2 + 2x - 4)</td>
</tr>
<tr>
<td>7. ((32x^7 + 28x^6 - 72x^3) ÷ (4x^4))</td>
<td>E. (x^2 + 3x + 5)</td>
</tr>
<tr>
<td>8. ((2x^3 - 13x^2 + 39x - 63) ÷ (2x - 7))</td>
<td>V. (x^2 - 3x + 9)</td>
</tr>
<tr>
<td>9. ((6x^4 + x^3 + 4x - 35) ÷ (3x + 5))</td>
<td>A. (2x + 5)</td>
</tr>
<tr>
<td>10. ((x^4 - 25) ÷ (x^2 - 5))</td>
<td>T. (x^2 + 5)</td>
</tr>
</tbody>
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**How Did the Light Dress Up for the Costume Party?**

Write a fraction (or 1) for each power. For each set of exercises, there is one extra answer. Write the letter of this answer in the corresponding box at the right.

<table>
<thead>
<tr>
<th>1</th>
<th>7⁻²</th>
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<th>6</th>
<th>10⁻¹</th>
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<td>2⁻⁷</td>
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<td>4⁻³</td>
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<td>D</td>
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<td>2⁻⁵</td>
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<td>C ¹/₅₁₂</td>
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<td>5⁻³</td>
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<td>P</td>
<td>¹/₁₄₄</td>
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<td>10</td>
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<td>¹/₆₉₃</td>
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<td>9⁻³</td>
<td>L ¹</td>
<td>I</td>
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**Answers**

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<tr>
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<td>V ¹/₁₂₈</td>
</tr>
<tr>
<td>10⁻⁵</td>
<td>I ¹/₂₅₆</td>
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<tr>
<td>¹/₆₄</td>
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<tr>
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<td>A ¹/₁₉₆</td>
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<tr>
<td>15⁻²</td>
<td>E ¹/₃₄₃</td>
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<td>¹/₃₀₀</td>
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<tr>
<td>11⁰</td>
<td>L ¹</td>
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<tr>
<td>16⁻¹</td>
<td>F ¹/₇₂</td>
<td></td>
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<tr>
<td>¹/₁₆</td>
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</table>
Why Did The Farmer Open A Bakery?

TO ANSWER THIS QUESTION: Express each product below as a single power of 10 or 8. Draw a straight line connecting each exercise with its answer. Each line will cross a number and a letter. The number tells you where to put the letter in the row of boxes at the bottom of the page.

\[ 10^4 \cdot 10^3 \]
\[ 10^{-4} \cdot 10^{-2} \]
\[ 10^6 \cdot 10^{-2} \]
\[ 8^{-4} \cdot 8^7 \]
\[ 8^{-1} \cdot 8^{-2} \]
\[ 8^{-5} \cdot 8^{-3} \]
\[ 10^2 \cdot 10 \]
\[ 10^{-2} \cdot 10^3 \]
\[ 10^{-5} \cdot 10^5 \]
\[ 8 \cdot 8^{-2} \]
\[ 8^{-7} \cdot 8^{-5} \]
\[ 8^{-6} \cdot 8^4 \]
\[ 10^3 \cdot 10^3 \]
\[ 10^{-8} \cdot 10 \]
\[ 10^4 \cdot 10^{-9} \]
\[ 8^{-6} \cdot 8^{-1} \]
\[ 8 \cdot 8 \]
\[ 8^4 \cdot 8^3 \]
Negative and zero exponents

Given a nonzero number \( a \) and a positive integer \( n \), the following definitions of negative exponents and zero exponents are stated.

1. For a negative exponent: the expression \( a^{-n} \) is the reciprocal of \( a^n \).

   This is written: \( a^{-n} = \frac{1}{a^n} \), where \( a \neq 0 \).

2. For a zero exponent: any nonzero number raised to the 0 power will have an answer of 1.

   This is written: \( a^0 = 1 \), where \( a \neq 0 \).

Evaluate \( 5^{-3} \)

Simplify by rewriting with positive exponents.

\[
\begin{align*}
5^{-3} &= \frac{1}{5^3} \\
&= \frac{1}{125}
\end{align*}
\]

\[
2x^{-3}y^2z^{-4} = 2 \cdot \frac{1}{x^3} \cdot y^2 \cdot \frac{1}{z^4}
\]

Note: When rewriting expressions to positive exponent form, simply move the factors from the denominator to the numerator or vice versa leaving out the in-between step.

1. Any number raised to the zero power has what value?

Rewrite each expression with positive exponents.

2. \( x^{-8} \)

3. \( \frac{1}{3x^2} \)

4. \( (-3)^0x^{-3} \)

5. \( x^{-10} \)

6. \( \frac{1}{5x^4} \)

7. \( \frac{6}{x^2} \)

8. \( 6x^{-3} \)

9. \( x^{-3}y^4 \)

10. \( \frac{1}{(2x)^3} \)

11. \( 4x^{-5} \)

12. \( 4x^4y^{-2} \)

13. \( (3x^{-2})^2 \)

Evaluate each expression.

14. \( 4^2 \)

15. \( 7^3 \cdot 7^{-3} \)

16. \( 3 \cdot 3^{-1} \)

17. \( -5^0 \cdot \frac{1}{3^3} \)

18. \( (6^2)^{-2} \)

19. \( (-2^{-3})^{-1} \)
Money no Matter

In 1997, NBC renewed its contract to produce a sitcom for the 1997–1998 season for a cost of $120 million—the most expensive TV series renewal ever recorded. This show also demanded the most money in television history for advertisement—more than $1 million per minute. What is the name of this famous show?

To find out, simplify each problem below using only positive exponents. Find your answers in the list provided and circle the corresponding letters. Unscramble the circled letters to discover the answer.

1. $2x^{-5}$
2. $-3x^2y^{-1}$
3. $2x^{-5}y^3$
4. $\frac{(7x)^0}{4x^0}$
5. $\frac{8x}{4^0x^4}$
6. $\frac{8x^3y^4}{-2xy^5}$
7. $\frac{-6x^4y^3}{2x^0y^3}$
8. $\left(\frac{6x^2y^3}{-2xy^4}\right)^0$

<p>| | | | | |</p>
<table>
<thead>
<tr>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>$\frac{-3y^6}{x^4}$</td>
<td>N</td>
<td>$\frac{2}{x^5}$</td>
<td></td>
</tr>
<tr>
<td>F</td>
<td>$\frac{2y^3}{x^5}$</td>
<td>E</td>
<td>$8x^5$</td>
<td></td>
</tr>
<tr>
<td>D</td>
<td>$\frac{-3}{y}$</td>
<td>O</td>
<td>$xy^4$</td>
<td></td>
</tr>
<tr>
<td>G</td>
<td>$-4xy$</td>
<td>E</td>
<td>$\frac{1}{4}$</td>
<td></td>
</tr>
<tr>
<td>R</td>
<td>$\frac{7}{4}$</td>
<td>B</td>
<td>$-9x^2y^7$</td>
<td></td>
</tr>
<tr>
<td>S</td>
<td>1</td>
<td>L</td>
<td>$\frac{-4y^9}{x^4}$</td>
<td></td>
</tr>
</tbody>
</table>

Answer: _________________
Division properties of exponents

When dividing exponents, it is important to remember the following properties:

1. When dividing powers that have the same base, subtract the exponents.
   
   For example, \( \frac{x^4}{x^2} = x^{4-2} = x^2 \), where \( x \) cannot be equal to 0.

2. When finding a power of a quotient, find the power of the numerator and the power of the denominator and divide.
   
   For example, \( \left( \frac{x}{y} \right)^3 = \frac{x^3}{y^3} \), where \( y \) cannot be equal to 0.

Simplify

\[
\frac{6^8}{6^6} = 6^{8-6} = 6^2 = 36
\]

Simplify

\[
\left( \frac{3}{4} \right)^2 = \frac{3^2}{4^2} = \frac{9}{16}
\]

1. Explain what you do with the exponents when dividing powers that have the same base.

Evaluate each expression.

2. \( \frac{5^6}{5^3} \)

3. \( \frac{(3y^2)^3}{3^2} \)

4. \( \frac{3^2}{3^2} \)

5. \( \frac{5^4 \cdot 5}{5^2} \)

6. \( \left( \frac{3}{2} \right)^3 \)

7. \( \frac{7^3}{7} \)

8. \( \frac{4^6}{4^8} \)

9. \( \frac{6^4 \cdot 6^3}{6^5} \)

10. \( \left( \frac{4}{5} \right)^2 \)

11. \( \left( \frac{-3}{4} \right)^2 \)

Simplify each expression.

12. \( \left( \frac{3}{x} \right)^3 \)

13. \( x^5 \cdot \frac{1}{x^7} \)

14. \( \frac{18x^4y^2}{-6x^2y^4} \cdot \frac{-3x^2y^2}{-y} \)

15. \( \frac{x^3}{x^5} \)

16. \( \frac{4x^4y^4}{4x^2y^2} \cdot \frac{4x^2y^4}{2xy} \)

17. \( \frac{7x^3y^2}{x^2y^3} \cdot \frac{(2x^3y)^2}{x^2y^2} \)

18. \( x^4 \cdot \frac{1}{x^2} \)

19. \( \frac{6x^2y^4}{3y^2} \cdot \frac{7x^2y^4}{x^4} \)

20. \( \frac{8x^3y^4}{x^2y^3} \cdot \frac{(4xy^2)^3}{x^2y^2} \)
Incredible Irony

Believe it or not, the people of Washington, D.C., did not always have the right to vote for the position of President of the United States. How many Presidents had the United States had before these people were allowed to vote?

To find out, simplify each term. Match your answers to those given. Write the letter corresponding to the solution above each problem number at the bottom of the page to spell out the answer.

1. \[ \frac{x^2y}{xy} \]
2. \[ \frac{-2x^2y}{18xy} \]
3. \[ \frac{10x^2y}{10x^2y} \]
4. \[ \frac{13xy}{26x^2y} \]
5. \[ \frac{(9y)^3}{9y} \]
6. \[ \frac{-18xy^2}{6x^2y^3} \]
7. \[ \frac{-12x^2y}{9xy^4} \]
8. \[ \frac{(-3xy)(6x^2y^4)}{9x^3y^2} \]
9. \[ \frac{(3x^2)(2xy)}{6xy} \]