**Complementary Angles**

Two angles are complementary angles if and only if the sum of their measure is $90^\circ$. Each angle is a complement of the other. A pair of angles do not need to be adjacent to be complementary.

In the figure above, $\angle AOB$ and $\angle BOC$ are a pair of complementary angles. $\angle AOB$ and $\angle D$ are also a pair of complementary angles. $\angle AOB$ is a complement of $\angle BOC$ and $\angle D$.

**Supplementary Angles**

Two angles are supplementary angles if and only if the sum of their measures is $180^\circ$. Each angle is a supplement of the other. Similar to complementary angles, supplementary angles do not need to be adjacent.

In the figure above, $\angle XOY$ and $\angle YOZ$ are supplementary angles. $\angle XOY$ and $\angle W$ are also supplementary angles. $\angle XOY$ is a supplement of $\angle YOZ$ and $\angle W$.

**Practice**

Find the measure of a complement of $\angle 1$ for each of the following measures of $\angle 1$.

Find the measure of a supplement of $\angle 2$ for each of the following measures of $\angle 2$.

1. $m\angle 1 = 44^\circ$  
2. $m\angle 1 = 72^\circ$  
3. $m\angle 1 = 80^\circ$  
4. $m\angle 1 = 25^\circ$  
5. $m\angle 1 = 5^\circ$  
6. $m\angle 2 = 88^\circ$  
7. $m\angle 2 = 130^\circ$  
8. $m\angle 2 = 60^\circ$  
9. $m\angle 2 = 115^\circ$  
10. $m\angle 2 = 1^\circ$
**VERTICAL ANGLES**

Vertical angles are two angles whose sides form two pairs of opposite rays. When two lines intersect, they form two pairs of vertical angles.

In the figure above, the two pairs of vertical angles are $\angle 1$ and $\angle 3$ and $\angle 2$ and $\angle 4$. Also, $\angle 1$ and $\angle 2$ are supplementary angles. Since $\angle 1$ and $\angle 3$ are both supplements to the same angle, they are congruent; in other words, they have the same measurement.

**PRACTICE**

Use the figure below to answer practice problems 11–13.

11. Name two angles that are supplements of $\angle DOE$.
12. Name a pair of complementary angles.
13. Name two pairs of vertical angles.

State whether the following statements are true or false.

14. Complementary angles must be acute.
15. Supplementary angles must be obtuse.
16. Two acute angles can be supplementary.
17. A pair of vertical angles can be complementary.
TRANSVERSALS

A transversal is a line that intersects two or more other lines, each at a different point. In the figure below, line \( t \) is a transversal, line \( s \) is not.

The prefix trans means to cross. In the figure above, you can see that line \( t \) cuts across the two lines \( m \) and \( n \). Line \( m \) is a transversal for lines \( s \) and \( t \). Also, line \( n \) is a transversal across lines \( s \) and \( t \). Line \( s \) crosses lines \( m \) and \( n \) at the same point (their point of intersection); therefore, line \( s \) is not a transversal. A transversal can cut across parallel as well as intersecting lines, as shown in the figure below:

PRACTICE

Use the figure below to answer questions 1–4.

1. Is line \( d \) a transversal? Why or why not?
2. Is line \( y \) a transversal? Why or why not?
3. Is line \( t \) a transversal? Why or why not?
4. Is line \( r \) a transversal? Why or why not?
PERPENDICULAR LINES
Perpendicular lines are another type of intersecting lines. Everyday examples of perpendicular lines include the horizontal and vertical lines of a plaid fabric and the lines formed by panes in a window. Perpendicular lines meet to form right angles. Right angles always measure 90°. In the following figure, lines x and y are perpendicular:

The symbol "⊥" means perpendicular. You could write, \( x \perp y \), to show these lines are perpendicular. Also, the symbol that makes a square in the corner where lines x and y meet indicates a right angle. In geometry, you shouldn't assume anything without being told. Never assume a pair of lines are perpendicular without one of these symbols. A transversal can be perpendicular to a pair of lines, but it does not have to be. In the figure below, line \( t \) is perpendicular to both line \( l \) and line \( m \).

PRACTICE
State whether the following statements are true or false.

5. Perpendicular lines always form right angles.

6. The symbol "⊥" means perpendicular.

7. Transversals must always be perpendicular.
NONINTERSECTING LINES
If lines do not intersect, then they are either parallel or skew.

Lines \( l \) and \( m \) are parallel.
Lines \( l \) and \( m \) are coplanar lines.
Lines \( l \) and \( m \) do not intersect.

Lines \( j \) and \( k \) are skew.
Lines \( j \) and \( k \) are noncoplanar lines.
Lines \( j \) and \( k \) do not intersect.

The symbol "\( l \parallel m \)" means parallel. So you can abbreviate the sentence "Lines \( l \) and \( m \) are parallel" by writing "\( l \parallel m \)." Do not assume a pair of lines are parallel unless it is indicated. Arrowheads on the lines in a figure indicate that the lines are parallel. Sometimes double arrowheads are necessary to differentiate two sets of parallel lines, as shown in the figure below:

Everyday examples of parallel lines include rows of crops on a farm and railroad tracks. An example of skew lines is the vapor trails of a northbound jet and a westbound jet flying at different altitudes. One jet would pass over the other, but their paths would not cross.

PRACTICE
Complete the sentences with the correct word: always, sometimes, or never.

8. Parallel lines are _______ coplanar.
9. Parallel lines _______ intersect.
10. Parallel lines are _______ cut by a transversal.
11. Skew lines are _______ coplanar.
ANGLES FORMED BY PARALLEL LINES AND A TRANSVERSAL

If a pair of parallel lines are cut by a transversal, then eight angles are formed. In the figure below, line \( l \) is parallel to line \( m \) and line \( t \) is a transversal forming angles 1–8. Angles 3, 4, 5, and 6 are inside the parallel lines and are called interior angles. Angles 1, 2, 7, and 8 are outside the parallel lines and are called exterior angles.

![Diagram of parallel lines and transversal]

ALTERNATE INTERIOR ANGLES

Alternate interior angles are interior angles on alternate sides of the transversal. In the figure above, angles 3 and 5 and angles 4 and 6 are examples of alternate interior angles. To spot alternate interior angles, look for a \( Z \)-shaped figure, as shown in the following figures:

![Diagrams of alternate interior angles]

SAME-SIDE INTERIOR ANGLES

Same-side interior angles are interior angles on the same side of the transversal. To spot same-side interior angles, look for a \( U \)-shaped figure in various positions, as shown in the examples below.

![Diagrams of same-side interior angles]
CORRESPONDING ANGLES

Corresponding angles are so named because they appear to be in corresponding positions in relation to the two parallel lines. Examples of corresponding angles in the figure below are angles 1 and 5, 4 and 8, 2 and 6, and 3 and 7.

To spot corresponding angles, look for an F-shaped figure in various positions, as shown in the examples below.

ANGLES FORMED BY NONPARALLEL LINES AND A TRANSVERSAL

Even when lines cut by a transversal are not parallel, you still use the same terms to describe angles, such as corresponding, alternate interior, and same-side interior angles. For example, look at the following figure:

∠1 and ∠5 are corresponding angles
∠3 and ∠5 are alternate interior angles
∠4 and ∠5 are same-side interior angles.
**POSTULATES AND THEOREMS**

As mentioned in Lesson 1, postulates are statements of fact that we accept without proof. Theorems are statements of fact that can be proved. You will apply both types of facts to problems in this book. Some geometry books teach formal proofs of theorems. Although you will not go through that process in this book, you will still use both postulates and theorems for applications. There are some important facts you need to know about the special pairs of angles formed by two parallel lines and a transversal. If you noticed that some of those pairs of angles appear to have equal measure, then you are on the right track. The term used for equal measure in geometry is congruent.

Another important fact you should know is that a pair of angles whose sum is 180° is called supplementary. Here are the theorems and the postulate that you will apply to a figure in the next few practice exercises.

**PRACTICE**

In the figure below, a∥b and c∥d. For questions 13–20, (a) state the special name for each pair of angles (alternate interior angles, corresponding angles, and same-side interior angles), then (b) tell if the angles are congruent or supplementary.

![Diagram of parallel lines and a transversal](image)

13. \( \angle 1 \) and \( \angle 9 \)
   a. \( \underline{\text{_____}} \) b. \( \underline{\text{_____}} \)

14. \( \angle 7 \) and \( \angle 11 \)
   a. \( \underline{\text{_____}} \) b. \( \underline{\text{_____}} \)

15. \( \angle 5 \) and \( \angle 10 \)
   a. \( \underline{\text{_____}} \) b. \( \underline{\text{_____}} \)

16. \( \angle 3 \) and \( \angle 11 \)
   a. \( \underline{\text{_____}} \) b. \( \underline{\text{_____}} \)

17. \( \angle 11 \) and \( \angle 14 \)
   a. \( \underline{\text{_____}} \) b. \( \underline{\text{_____}} \)

18. \( \angle 8 \) and \( \angle 12 \)
   a. \( \underline{\text{_____}} \) b. \( \underline{\text{_____}} \)

19. \( \angle 9 \) and \( \angle 11 \)
   a. \( \underline{\text{_____}} \) b. \( \underline{\text{_____}} \)

20. \( \angle 8 \) and \( \angle 16 \)
   a. \( \underline{\text{_____}} \) b. \( \underline{\text{_____}} \)
Triangles

Naming Triangles by Side Lengths

**Scalene**
- All sides have different lengths.

**Isosceles**
- Two sides have the same length.

**Equilateral**
- All sides have the same length.

Naming Triangles by Angle Measures

**Acute**
- All angles have measures less than 90°.

**Right**
- One angle has a measure of 90°.

**Obtuse**
- One angle has a measure of more than 90°.

Write the name of each triangle according to the measure of its angles.

1. \[\triangle \text{ with } 35°, 115°, 30°\]  
2. \[\triangle \text{ with } 80°, 20°, 50°\]  
3. \[\triangle \text{ with } 90°, 40°, 40°\]  
4. \[\triangle \text{ with } 15°, 150°, 15°\]

Write the name of each triangle according to the length of its sides.

5. \[\triangle \text{ with sides } 12 \text{ cm, 12 cm, 12 cm}\]  
6. \[\triangle \text{ with sides } 18 \text{ cm, 18 cm, 5 cm}\]  
7. \[\triangle \text{ with sides } 18 \text{ cm, 18 cm, 16 cm}\]  
8. \[\triangle \text{ with sides } 3 \text{ cm, 7 cm, 7 cm}\]

Find the degree measure \(x\) in each triangle.

9. \[\triangle \text{ with } 95°, 50°, \, ? \quad \text{or } \quad 95° + 50° + ? = 180°
\]
\[x = \quad \text{or } \quad x = \quad \text{or } \quad x = \quad \]

10. \[\triangle \text{ with } 90°, 48°, \, ? \quad \text{or } \quad 90° + 48° + ? = 180°
\]
\[x = \quad \text{or } \quad x = \quad \text{or } \quad x = \quad \]

11. \[\triangle \text{ with } 112°, 36°, \, ? \quad \text{or } \quad 112° + 36° + ? = 180°
\]
\[x = \quad \text{or } \quad x = \quad \text{or } \quad x = \quad \]
Circles

A circle is all the points in a plane that are the same distance from one point called the center.

Use the circle $P$ for exercises 1–8.

Write the name for each part of the figure.

1. $\overline{RU}$
2. $\overline{PS}$
3. $\overline{TU}$
4. $\overline{WX}$
5. $\overline{RS}$
6. $\angle SPR$
7. $\overline{UM}$
8. $\overline{WX}$
9. $P$
10. $\overline{RUS}$
11. shaded area enclosed by $\angle RPS$
12. shaded area cut off by $WX$
Glass Gallant

On May 18, 1996, Ashrita Furman of Jamaica, New York, made the Guinness Book of World Records when he balanced pint glasses from his chin for 11.89 seconds. How many of these pint glasses did he balance?

To find out, use the Pythagorean theorem to solve for the missing value in each problem. (Round decimals to the nearest hundredth.) Match each set of values in Column A to its matching solution in Column B. Read down the column of written letters to reveal the answer.

<table>
<thead>
<tr>
<th>Column A</th>
<th>Column B</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. $a = 8, c = 17, b =$</td>
<td>S. 9.54</td>
</tr>
<tr>
<td>2. $a = 6, b = 2, c =$</td>
<td>E. 9.80</td>
</tr>
<tr>
<td>3. $a = 6, b = 8, c =$</td>
<td>F. 10</td>
</tr>
<tr>
<td>4. $b = 20, c = 29, a =$</td>
<td>I. 6.32</td>
</tr>
<tr>
<td>5. $a = 7, c = 25, b =$</td>
<td>E. 30</td>
</tr>
<tr>
<td>6. $a = 3, c = 10, b =$</td>
<td>N. 40</td>
</tr>
<tr>
<td>7. $b = 40, c = 50, a =$</td>
<td>Y. 24</td>
</tr>
<tr>
<td>8. $a = 1, b = 3, c =$</td>
<td>F. 15</td>
</tr>
<tr>
<td>9. $b = 5, c = 11, a =$</td>
<td>V. 3.16</td>
</tr>
<tr>
<td>10. $a = 9, c = 41, b =$</td>
<td>T. 21</td>
</tr>
</tbody>
</table>

Answer: ____________________________
Quadrilaterals come in a great many varieties. Several types are shown here.

<table>
<thead>
<tr>
<th>Squares</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rhombuses (or rhombi)</td>
</tr>
<tr>
<td>Rectangles</td>
</tr>
<tr>
<td>Parallelograms</td>
</tr>
<tr>
<td>Kites</td>
</tr>
<tr>
<td>Trapezoids</td>
</tr>
<tr>
<td>No special names, just quadrilaterals</td>
</tr>
</tbody>
</table>
PROPERTIES OF PARALLELOGRAMS

The following properties of parallelograms will help you to determine if a figure is a parallelogram or just a quadrilateral. The properties are also useful to determine measurements of angles, sides, and diagonals of parallelograms.

**Properties of Parallelograms**

**Definition:** Opposite sides of a parallelogram are parallel.

**Theorem:** Opposite sides of a parallelogram are congruent.

**Theorem:** Opposite angles of a parallelogram are congruent.

**Theorem:** Consecutive angles of a parallelogram are supplementary.

**Theorem:** Diagonals of a parallelogram bisect each other.
Be aware that diagonals of a parallelogram are not necessarily congruent. Watch out for this, because many students make this common error.

Examples:

\[ \overline{BM} = 3 \text{ Opposite sides are congruent.} \]
\[ \overline{BH} = 5 \text{ Opposite sides are congruent.} \]
\[ \angle M = 135^\circ \text{ Opposite angles are congruent.} \]
\[ \angle A = 45^\circ \text{ Consecutive angles are supplementary.} \]
\[ \angle B = 45^\circ \text{ Opposite angles are congruent.} \]

\[ \angle XWZ = 90^\circ + 45^\circ = 135^\circ \text{ Angle Addition Postulate} \]
\[ \angle XYZ = 135^\circ \text{ Opposite angles are congruent.} \]
\[ \angle WXY = 45^\circ \text{ Consecutive angles are supplementary.} \]
\[ \angle WZY = 45^\circ \text{ Opposite angles are congruent.} \]
\[ \overline{WO} = 2 \text{ Diagonals bisect each other.} \]
\[ \overline{ZO} = 6 \text{ Diagonals bisect each other.} \]
**QUADRILATERALS**

**PRACTICE**

Use the figure below to find each side length and angle measure for questions 11–15.

[Diagram of a parallelogram with sides labeled A, B, C, D and angles 60° and 90°]

11. \(BC\)
12. \(DC\)
13. \(\angle B\)
14. \(\angle A\)
15. \(\angle C\)

Use the figure below to find each side length and angle measure for questions 16–20.

[Diagram of a quadrilateral with sides labeled P, Q, S, R and angles 20° and 45°]

16. \(SQ\)
17. \(OR\)
18. \(\angle PQR\)
19. \(\angle SPQ\)
20. \(\angle SRQ\)
OTHER SPECIAL PROPERTIES

There are a few other special properties for the rectangle, rhombus, and square. First, remember that these figures are all parallelograms; therefore, they possess the same properties of any parallelogram. However, because these figures are special parallelograms, they also have additional special properties. Since a square is both a rectangle and a rhombus, a square possesses these same special properties.

**Theorem:** The diagonals of a rectangle are congruent.

**Theorem:** The diagonals of a rhombus are perpendicular, and they bisect the angles of the rhombus.

Examples:

\[ \overline{XE} = 6 \text{ Diagonals bisect each other.} \]

\[ \overline{NW} = 12 \text{ Diagonals of a rectangle are congruent.} \]

\[ \angle NES = 20^\circ \text{ Alternate interior angles are congruent, when formed by parallel lines.} \]

\[ \angle NSW = 90^\circ \text{ Definition of a rectangle.} \]

\[ \angle BEC = 90^\circ \text{ Diagonals of a rhombus are perpendicular.} \]

\[ \angle DCE = 30^\circ \text{ Diagonals of a rhombus bisect the angles.} \]

\[ \angle BAD = 60^\circ \text{ Opposite angles are congruent.} \]

\[ \angle ABC = 120^\circ \text{ Consecutive angles are supplementary.} \]

\[ \angle ADC = 120^\circ \text{ Opposite angles are congruent.} \]
Practice

Use the figure below to find the side length and angle measures for questions 21–24.

21. \(\overline{QS}\)
22. \(\overline{OP}\)
23. \(\angle QSR\)
24. \(\angle RPS\)

Use the figure below to find the angle measures for questions 25–30.

25. \(\angle JKM\)
26. \(\angle JML\)
27. \(\angle KJM\)
28. \(\angle KLM\)
29. \(\angle JNK\)
30. \(\angle KNL\)