Math 110

Applications of Exponential Functions

The balance $A$ produced by a principal $P$ (original investment or loan), with an annual interest rate $r$ (in decimal form), compounded $n$ times per year, after $t$ years, is given by:

$$A = P \left(1 + \frac{r}{n}\right)^{nt}$$

This is known as the Compound Interest Formula.

ex. If you borrow $20,000 with an annual interest rate of 6% compounded monthly, how much do you owe after 3 years?

ex. Assume that today, you invest $1,000 at an annual interest rate of 3.5% compounded quarterly. How long will it take for your investment to reach $1,500?
When the number of compoundings increases with no bound, our compound interest formula becomes:

\[ A = Pe^{rt} \]

This is called the Continuous Compounding Interest Formula.

ex. How long will it take $4,500 to double if you invest it at an annual interest rate of 5% compounded continuously?
An initial population $N_0$ grows into a population $N(t)$ after $t$ years, according to the exponential growth formula $N(t) = N_0 e^{kt}$, where $k$ is a positive growth constant.

ex. A town had a population of 53,700 in 1996 and a population of 58,100 in 2000.

a) Find the exponential growth function for this town. Use $t = 0$ to represent 1996.

b) Use your function above to predict this town’s population in 2008.
An initial amount of radioactive material $N_0$ decreases into the amount $N(t)$ after $t$ years, according to the exponential decay formula $N(t) = N_0 e^{kt}$, where $k$ is a negative decay constant.

ex. Polonium has a half-life of 138 days.

a) Find the exponential decay function for the amount of Polonium remaining in a sample after $t$ days.

b) Use your function above to predict how long it will take 10 kg of Polonium to be reduced to 0.2 g.
Logarithmic Applications

The expiration time for the world’s oil supply is modeled by the equation

\[ T = 14.29 \ln(0.00411r + 1), \]

where \( r \) is the estimated world oil reserves in billions of barrels of oil and \( T \) is the time before that amount is depleted. How many barrels of oil are necessary to last 50 years?