Appendix B
Metric System

Introduction

Measurement systems are essential for human activities. For economic trade measurement systems provide us with confidence that all parties involved are referring to the same quantity of materials being traded. When your doctor writes a prescription a pharmacist is able to precisely measure the exact quantity of the drug your doctor intended for you to receive. Without measurement systems economic trade would be paralyzed and our ability to communicate with one another would be greatly diminished.

When using a measurement system it is very common to convert between units within the measurement system. For example, carpenters frequently convert measurements made in inches to feet and gamblers who play dime slot machines have their dollars converted to dimes. If you ask your bank to convert a ten dollar bill to quarters it is obviously to your advantage to know how to make this conversion so you can insure you receive the correct number of quarter.

The standardized measurement system used in science is the metric system. The metric system was developed in France in 1791 and today it is the most widely used measurement system in the world. The metric system is one of the easiest measurement systems to use because it is a decimal system, meaning all of its parts (units) are based on fractions or multiples of ten, such as . . . 1000 100 10 1 0.1 0.01 0.001 . . . . Table 2 shows the most common decimal multiples and fractions used in the metric system. It is much easier to convert between units within a decimal system than it is within a nondecimal system. Consider the following examples showing conversions within a decimal and a nondecimal system.

<table>
<thead>
<tr>
<th>Decimal Systems:</th>
<th>4.00 = ______ cents</th>
<th>(Answer: 400 cents)</th>
</tr>
</thead>
<tbody>
<tr>
<td>42 cm = ______ mm</td>
<td></td>
<td>(Answer: 420 mm)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Nondecimal system:</th>
<th>3 feet = ______ inches</th>
<th>(Answer: 36 inches)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.2 pounds = ______ ounces</td>
<td></td>
<td>(Answer: 19.2 ounces)</td>
</tr>
</tbody>
</table>

Notice that when converting between units within a decimal system (e.g. dollars to cents or cm to mm) you only have to move the decimal point. However, when converting within a nondecimal system you can’t simply move the decimal point, instead you have to first know the conversion (e.g. 12 inches in a foot, or 16 ounces in a pound) and then multiple or divide the number to be converted by the proper conversion. Who then has the legitimate complaint, a foreign student who grew up using the metric system being
asked to learn the nondecimal English system (e.g. inches and feet, ounces and pounds) or the American student being asked to learn the metric system? Decimal systems rule!

The modern metric system has been renamed the International System of Units (denoted by the letters SI). SI was established in 1960 at the 11th General Conference on Weights and Measures. It was then that metric units, definitions, and symbols were revised, simplified and standardized. There are two basic parts of the metric system that you need to know: the metric base units (Table 1) and the metric prefixes (Table 2).

Table 1. Some of the base units of the SI system (top table) and a comparison of base units in the SI and the English system (bottom table).

<table>
<thead>
<tr>
<th>Physical Quantity</th>
<th>Name of SI unit</th>
<th>Symbol for SI unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>length</td>
<td>meter</td>
<td>m</td>
</tr>
<tr>
<td>mass</td>
<td>kilogram</td>
<td>kg</td>
</tr>
<tr>
<td>volume</td>
<td>liter</td>
<td>l</td>
</tr>
<tr>
<td>temperature</td>
<td>Celsius*</td>
<td>C</td>
</tr>
<tr>
<td>amount of substance</td>
<td>mole**</td>
<td>mol</td>
</tr>
</tbody>
</table>

*Water freezes at 0°C (32°F) and boils at 100°C (212°F).
Converting Fahrenheit to Celsius:  \( \frac{9}{5}C = F - 32; \quad F = \frac{9}{5}C + 32; \quad C = \frac{5}{9}(F-32) \)
**A mole contains 6.20 x 10²³ atoms or molecules

Comparisons between Metric and English units

<table>
<thead>
<tr>
<th>Physical Quantity</th>
<th>Metric (SI) System</th>
<th>English (US) System</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Length</strong></td>
<td>1 meter</td>
<td>39.37 inches</td>
</tr>
<tr>
<td><strong>Volume</strong></td>
<td>1 liter</td>
<td>1.0567 liquid quarts</td>
</tr>
<tr>
<td><strong>Weight</strong></td>
<td>1 gram</td>
<td>0.035 ounces (½ of a dime)</td>
</tr>
<tr>
<td><strong>Temperature</strong></td>
<td>37°C</td>
<td>98.6°F</td>
</tr>
</tbody>
</table>

*1 gram of water at 4°C is equal to 1 cubic centimeter (cc or cm³) and is equal to one ml.

Questions: Using the information in Table 1 and Table 2 answer the following questions. Two of the questions have already been answered for you.

Given either the name or the symbol of the prefix, give the other:

1) c  centi
2) k
3) da
4) µ
5) d
Given the prefix size, give its name:

8) $10^{-6}$  \[ \text{µ} \]
9) $10^6$  \[ \text{____} \]
10) 1,000  \[ \text{____} \]

**Table 2.** The prefixes for the SI system. In naming the prefixes Greek was used for naming multipliers (prefixes greater than 10) and Latin was used for naming fractions (prefixes less than 1). Each metric prefix (e.g. centi) modifies the root word it follows (e.g. meter). For example, the prefix “centi” refers to $1/100$ (0.01). A centigram means $1/100$ of a gram and a centihamburger = $1/100$ of a hamburger. In centigram (cg), centi is the prefix and gram is the root (base unit). You do *not* need to know (memorize) the prefixes that are shaded in the following table.

<table>
<thead>
<tr>
<th>Prefix</th>
<th>Symbol</th>
<th>Multipliers</th>
<th>Exponential</th>
</tr>
</thead>
<tbody>
<tr>
<td>yotta</td>
<td>Y</td>
<td>1E+24</td>
<td>1024</td>
</tr>
<tr>
<td>zetta</td>
<td>Z</td>
<td>1E+21</td>
<td>1021</td>
</tr>
<tr>
<td>exa</td>
<td>E</td>
<td>1E+18</td>
<td>1018</td>
</tr>
<tr>
<td>peta</td>
<td>P</td>
<td>1E+15</td>
<td>1015</td>
</tr>
<tr>
<td>tera</td>
<td>T</td>
<td>1E+12</td>
<td>1012</td>
</tr>
<tr>
<td>giga</td>
<td>G</td>
<td>1000000000</td>
<td>109</td>
</tr>
<tr>
<td>mega</td>
<td>M</td>
<td>1000000</td>
<td>106</td>
</tr>
<tr>
<td>kilo</td>
<td>K</td>
<td>1000</td>
<td>103</td>
</tr>
<tr>
<td>hecto</td>
<td>H</td>
<td>100</td>
<td>102</td>
</tr>
<tr>
<td>deca</td>
<td>D</td>
<td>10</td>
<td>101</td>
</tr>
<tr>
<td>Standard unit</td>
<td></td>
<td>1</td>
<td>100</td>
</tr>
<tr>
<td>deci</td>
<td>d</td>
<td>0.1</td>
<td>$10^{-1}$</td>
</tr>
<tr>
<td>centi</td>
<td>c</td>
<td>0.01</td>
<td>$10^{-2}$</td>
</tr>
<tr>
<td>milli</td>
<td>m</td>
<td>0.001</td>
<td>$10^{-3}$</td>
</tr>
<tr>
<td>micro</td>
<td>µ</td>
<td>0.000001</td>
<td>$10^{-6}$</td>
</tr>
<tr>
<td>nano</td>
<td>n</td>
<td>0.000000001</td>
<td>$10^{-9}$</td>
</tr>
<tr>
<td>pico</td>
<td>p</td>
<td>1E-12</td>
<td>$10^{-12}$</td>
</tr>
<tr>
<td>femto</td>
<td>f</td>
<td>1E-15</td>
<td>$10^{-15}$</td>
</tr>
<tr>
<td>atto</td>
<td>a</td>
<td>1E-18</td>
<td>$10^{-18}$</td>
</tr>
</tbody>
</table>
Making Metric Conversion

Throughout the semester you will be asked to make metric conversions. When making metric conversions today, and during the rest of the semester, you will have to work with sign numbers, exponents, scientific notation and significant figures. If you have not learned, or if you have forgotten, how to work with sign numbers, exponents, scientific notation and/or significant figures refer to Appendix A for a brief tutorial.

There are several ways to successfully complete metric conversions, three common methods are outlined below.

1. **Method 1: Memorization Method – Simple, Yet Slow**
   This method should be used only if you are confused by the other two methods described below because the other two methods are faster to use (once they are learned) and they are more commonly used.

   **Step 1.** You must know the metric system prefixes and be able to arrange them in a descending order of magnitude. An easy way to memorize part of the descending sequence is “King Henry Drank Milk During Christmas Mass.” Look at the following descending arrangement of the metric prefixes. The two dashes between giga and mega, between mega and kilo and milli and micro are important and cannot be excluded. Table 2 shows the value of each metric prefix.

<table>
<thead>
<tr>
<th>Prefix</th>
<th>Symbol</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>zepto</td>
<td>z</td>
<td>$10^{-21}$</td>
</tr>
<tr>
<td>yocto</td>
<td>y</td>
<td>$10^{-24}$</td>
</tr>
</tbody>
</table>

   **Step 2.** Find the starting point and ending point for the conversion you are trying to make. For example, if you are asked to convert 43.2 cg to kg your starting point is “c” (cg) and your ending point is “k” (kg).

   **Step 3.** Count the decimal jumps from the starting unit to the ending unit as shown in Figure 1. The first jump in our example (43.2 cg to kg) is from “c” to “d.” Do not count “c” as a jump (which is a common error). Also, determine the
direction (left or right) in which you are jumping.

**Step 4.** Move the decimal in the direction and the number of jumps determined in Step 3 above. The correct answer becomes 0.000432 kg.

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**Figure 1.** An example showing how to make a metric conversion using Method 1. In this example you are asked to convert 43.2 cg into kilograms (kg). The correct answer is that 43.2 cg is equal to 0.000432 kg.

This “simple” method is not a fancy trick. When the prefixes are arranged in a descending order of magnitude (e.g. k h da m/l/g d c m) each decimal jump to the left is a division by 10 and each decimal jump to the right is a multiplication by 10. In the example (43.2 cg to kg) you made 5 jumps to the left, which is the same as dividing the original number (43.2) by 100,000 (or 10x10x10x10x10). If you happened to knew that 1 kg = 100,000 cg you could have simply divided 43.2 cg by 100,000 (43.2/100,000 = 0.000432).

**Problems:** Using Method 1 complete the following metric conversions.

1. 1 m = _________ mm
2. 33.5 kl = _________ dl
3. \(0.45 \, \mu g = \ \underline{\hspace{2cm}} \, g\)
4. \(3212.23 \, cg = \ \underline{\hspace{2cm}} \, Mg\)

\textit{Answers:} \(1000, \ 335000, \ 0.00000045, \ 0.0000321223\)

2. \textbf{Methods 2 & 3: Using the exponential differences between units}

Using these methods to convert between metric units you’ll need to know:

1) the names and symbols for the metric prefixes (Table 2)
2) which of the two prefixes represents a larger amount (Table 2)
3) the exponential "distance" between the two prefixes (Table 2)
4) scientific notation (see Appendix A)

These methods for making metric conversions require you to know the exponential distance between the two prefixes in the conversion you are attempting to make. For example, the exponential distance between milli (\(10^{-3}\)) and centi (\(10^{-2}\)) is \(10^{-1}\) (10), such that 1 cm = 10 mm and 1 mm = 0.1 cm. The exponential difference between kilo(\(10^3\)) and mega (\(10^6\)) is \(10^3\) (1000), such that 1 Mg = 1000 Kg and 1 Kg = 0.001 Mg.

You will need to compare the two exponents and determine the exponential differences between them. If you were asked to convert kilometers to centimeters you would:

1. Determine the value of each prefix. Kilo = 1000 (or \(10^3\)); centi = 0.01 or (\(10^{-2}\))
2. Then you would determine the exponential difference between them, which is \(10^3 - 10^{-2}\). You can either count the decimal jumps to get this difference (as in “Method 1” described above) or you could subtract the exponents. As described in Appendix A, to subtract these exponents you would subtract as follows: \(3-(-2) = 3+2 = 5\).
   If you looked at a number line from -2 to 3 you’d see: -2 -1 0 1 2 3 (and you’d count five jumps going from one end to the other).
3. Using this difference (i.e. 5) you’d either move the decimal place 5 places or change the exponent by 5. These steps will be described below.

Compute the exponential distance between milli and centi. \underline{__________}

Compute the exponential distance between kilo and centi. \underline{__________}

Compute the exponential distance between meter and km \underline{__________}

\textit{(Answers:} \(10 \ or \ 10^1; \ 100,000 \ or \ 10^5; \ 1000 \ or \ 10^3)\)
**Rule to follow:** Notice that when the two prefixes have the same sign (two negatives or two positives), to determine the exponential difference between them you subtract the two exponents (ignore the signs). For example, the difference between centi \((10^{-2})\) and milli \((10^{-3})\) is \(10^1\) \((3 - 2 = 1)\). If the two prefixes have different signs (i.e. one is positive and one is negative), to determine the exponential difference between them you add the two exponents (ignore the sign). For example, the difference between centi \((10^{-2})\) and kilo \((10^3)\) is \(10^5\) \((2 + 3 = 5)\).

**Method 2: Adding or subtracting the exponential difference**

In this method you simply use the exponential difference between the starting and ending unit (prefix) to make the metric conversion (as described above).

**Step 1:** Determine the exponent of the starting and the ending unit (prefix). For example, if you were converting 43.2 cg to kg you should know that “c” is \(10^{-2}\) and “k” is \(10^3\). Refer to Table 2 for this information.

**Step 2:** Subtract the starting unit (e.g. \(10^{-2}\) for “c”) from the ending unit (\(10^3\) for “k”) Be certain to determine if the difference is positive or negative. For example, going from cg to kg the difference is negative: \((-2)-(3) = -5\) going from kg to cg the difference is positive: \((3)-(2) = +5\)

**Option 1:** If you are using normal notation (e.g. 43.2 cg) you simply move the decimal point by the calculated difference in step 2 above. If the difference was **positive** move the decimal point to the right. If the difference was **negative** move the decimal point to the left.

**Example:** 43.2 cg = ____ kg
Since the exponential difference is - 5, move the decimal 5 places to the left. Therefore, the answer is 0.000432 kg.

**Option 2:** If you are using scientific notation (e.g. 4.32 \(\times 10^1\) cg) you simply add or subtract the calculated difference to the exponent. For example, if our original problem (i.e. convert 43.2 cg to kg) was in scientific notation it would have been written “convert \(4.32 \times 10^1\) cg to kg.” If the exponent difference is **positive** add the difference to the starting exponent. If the exponent difference is **negative** subtract the difference from the starting exponent.

**Example:** \(4.32 \times 10^1\) cg = ____ kg
Since the exponential difference is -5, subtract 5 from the original exponent. Therefore, the answer is $4.32 \times 10^{-4}$ kg (calculated as: $10^1 - 10^{-5} = 10^{-4}$)

Notice that Option 1 and Option 2 give the same answer: $0.000432$ kg = $4.32 \times 10^{-4}$ kg.

Problems:

1. Using the method outlined in Option 1 convert 0.985 ml to meters.
2. Using the method outlined in Option 2 convert $3.56 \times 10^4$ ml to meters.

(Answers: 0.000985, $3.56 \times 10^4$)

Method 3: Conversion factors (Dimensional Analysis)

Dimensional Analysis (also called the Unit Factor Method) is a problem-solving method that uses the fact that any number or expression can be multiplied by one without changing its value. This is a very useful technique and it is the most method used to make metric conversion. For example, because 1 liter = 1000 mL, $1\text{L}/1000\text{mL}$ is equivalent to 1 (in the same way that 12/1 dozen is equivalent 1). If you were ask to convert 50.0 mL to liters you could solve this as follows:

(by simply multiplying $50 \text{ ml} \times 1$ (since $1\text{L}/1000\text{ml} = 1$; which when solved = $50/1000 = 0.0500 \text{ L}$)).

$$\frac{? \text{L}}{1000 \text{ mL}} = \frac{50.0 \text{ mL}}{1000 \text{ mL}} \times \frac{1 \text{ L}}{1000 \text{ mL}} = 0.0500 \text{ L}$$

The logic behind this method (and example) is as follows:

1. First cancel out mL and convert to liters (L). To do this multiply $50.0 \text{ mL}$ by another fraction (called a conversion factor) in which you place mL in the denominator and liters (L) in the nominator.
2. The fraction (conversion factor) your are multiplying $50.0 \text{ mL}$ by must be equivalent to one. You make L/mL equivalent to one in two ways, either as $1\text{L}/1000\text{mL}$ or as $0.001\text{L}/1\text{ml}$. Obviously in the example we elected to use $1\text{L}/1000\text{mL}$, although $0.001\text{L}/1\text{ml}$ would have given us the exact same answer.
3. Now you simply multiple 50 times 1/1000 to get the correct answer, $0.0500\text{L}$
Let’s try an example – using this method convert 43.2 cg into dg. We need to find a conversion factor that would cancel out cg and replace cg with dg. The partial answer is dg/cg because if we multiplied cg x dg/cg we get dg. The complete conversion factor would be 1dg/10cg because this is the equivalent relationship between g and cg. When we multiply 43.2 cg x 1dg/10cg we end up with the correct answer, which is 4.32 dg. In this example another correct conversion factor would have been 0.1 dg/1cg. Why? (answer: 43.2 cg x 0.1dg/1cg = 4.32dg, just as 43.2 cg x 1dg/10cg = 4.32 dg) (1dg = 10cg, and 0.1 dg = 1 cg) (just as $1 = 10 dimes, or 1 dime = 0.1 $)

Try another example, this time using money. Convert $1.23 to cents.  

\[
\begin{align*}
\$1.23 & \times \frac{10 \text{ dimes}}{1 \text{ \$}} \times \frac{10 \text{ cents}}{1 \text{ dime}} = 123 \text{ cents} \\
\end{align*}
\]

In the first step we convert dollars to dimes and in the second step we convert dimes to pennies. As long as each conversion factor is equivalent to one we will not change the true value of the money.

Problems

1. Using conversion factor convert 0.080 cm to Km

   Answer using normal notation:

   \[
   = \frac{0.080 \text{ cm}}{1 \text{ cm}} \times \frac{1 \text{ km}}{100000 \text{ cm}} = 0.000008 \text{ km}
   \]

   Answer using exponents:

   \[
   = \frac{0.080 \text{ cm}}{1 \text{ cm}} \times \frac{1 \text{ km}}{10^5 \text{ cm}} = 8 \times 10^{-2} \times 10^{-(5)} = 8 \times 10^{-7}
   \]

   Recall the formula for dividing exponents: \(a^m/a^n = a^{m-n}\)

2. Using conversion factors, make the following conversions:

   a. 0.75 kg to milligrams
   b. 1500 millimeters to km
   c. 2390 g to kg
   d. 0.52 km to meters
   e. 65 kg to g

   (Answers: 7.5 x 10^5 mg, 1.5 x 10^{-3} km, 2.39 kg, 520 m, 6.5 x 10^4 g )
If you make some mistakes doing metric conversions you are not alone. Consider the $125 million error made by experienced rocket scientists (see below).

**Metric mishap caused loss of NASA orbiter**

*September 30, 1999*

*By Robin Lloyd  CNN Interactive Senior Writer*

(CNN) -- NASA lost a $125 million Mars orbiter because a Lockheed Martin engineering team used English units of measurement while the agency's team used the more conventional metric system for a key spacecraft operation, according to a review finding released Thursday. It took four years to build the Mars Climate Orbiter and 10 months for it to travel 600 million miles to its humiliating fiery death on the red planet. One grammar school mistake destroyed it all.