Hypothesis Testing for a Mean when $\sigma$ is known

If a preliminary study or other information gives us an idea about the value of $\sigma$, then the hypothesis testing procedure for a mean is as follows.

**Assumptions:**
- Population has a normal distribution OR $n \geq 30$
- $\sigma$ is known.
- A simple random sample of size $n$ is taken from the population.

**Steps**
1. State $H_0$ and $H_1$.
2. Compute the standardized test statistic: $z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}$.
3. Use the standard normal distribution and the type of test (left-tailed, right-tailed or two-tailed) to find the P-value corresponding to the test statistic. Make sure you draw in the curve.
4. Conclude the test. If the P-value $\leq \alpha$, then reject $H_0$. If the P-value $> \alpha$, then do not reject $H_0$.
5. Interpret your conclusion in the context of the problem (complete sentences).

**Example 1**
Gentle Ben is a Morgan horse at a Colorado ranch. Over the past 8 weeks, a veterinarian took the following glucose readings from this horse (in mg/100 ml).

<table>
<thead>
<tr>
<th>Glucose Reading (mg/100 ml)</th>
</tr>
</thead>
<tbody>
<tr>
<td>93</td>
</tr>
<tr>
<td>88</td>
</tr>
<tr>
<td>82</td>
</tr>
<tr>
<td>105</td>
</tr>
<tr>
<td>99</td>
</tr>
<tr>
<td>110</td>
</tr>
<tr>
<td>84</td>
</tr>
<tr>
<td>89</td>
</tr>
</tbody>
</table>

The sample mean is $\bar{x} \approx 93.8$ . Let $x$ be a random variable representing glucose readings taken from Gentle Ben. We may assume that $x$ has a normal distribution and we know from past experience that $\sigma = 12.5$. The mean glucose levels for horses should be $\mu = 85 \text{mg/100ml}$. Do these data indicate that Gentle Ben has an overall average glucose level higher than 85? Use $\alpha = 0.05$.

1. $H_0$: $\mu \leq 85$
2. $H_1$: $\mu > 85$
3. $z = \frac{\bar{x} - \mu}{s/\sqrt{n}}$
4. Find the P-value:
5. Conclusion:
6. In words:
Example 2

Total blood volume (in ml) per body weight (in kg) is important in medical research. For healthy adults, the red blood cell volume mean is about $\mu = 28 \text{ ml/kg}$. Red blood cell volume that is too low or too high can indicate a medical problem. Suppose that Roger has had seven blood tests and his red blood cell volumes were

\[32 \quad 25 \quad 41 \quad 35 \quad 30 \quad 37 \quad 29\]

The sample mean is $\bar{x} \approx 32.7 \text{ ml/kg}$.

Let $x$ be a random variable that represents Roger’s red blood cell volume. Assume that $x$ has a normal distribution and that $\sigma = 4.75$. Use a 0.01 level of significance. Do the data indicate that Roger’s red blood cell volume is different (either way) from $\mu = 28 \text{ ml/kg}$?

1. $H_0$: $H_1$: 
2. $z =$
3. Find the $P$-value:
4. Conclusion:
5. In words:

Hypothesis Testing for $\mu$ when $\sigma$ is unknown

Assumptions:
- Population has a normal distribution or is mound shaped and symmetric. If you cannot assume this, then $n \geq 30$
- $\sigma$ is unknown.
- A simple random sample of size $n$ is taken from the population.
Steps

1. State $H_0$ and $H_1$.
2. Compute the sample test statistic: $t = \frac{\bar{x} - \mu}{s/\sqrt{n}}$ with $df = n - 1$.
3. Use the t-distribution and the type of test (left-tailed, right-tailed or two-tailed) to find the P-value corresponding to the test statistic. Make sure you draw in the curve.
4. Conclude the test. If the P-value $\leq \alpha$, then reject $H_0$. If the P-value $> \alpha$, then do not reject $H_0$.
5. Interpret your conclusion in the context of the problem (complete sentences).

Example 3

Pyramid Lake is in the Pauite Indian Reservation in Nevada. The lake is famous for its cutthroat trout. It is said that the average length of trout caught in Pyramid Lake is $\mu = 19$ inches. However, the Creel Survey reported that of a random sample of 51 fish caught, the mean length was $\bar{x} = 18.5$ inches with $s = 3.2$ inches. Do these data indicate that the average length of a trout caught in Pyramid lake is less than $\mu = 19$ inches? Use a 5% level of significance.

1. $H_0$: $\mu = 19$ inches
   $H_1$: $\mu < 19$ inches

2. $t =$

3. Find the P-value:

4. Conclusion:

5. In words:

Note: for part 3, use tcdf(left, right, df)