Math 110 – Chapter 4 – Worksheet 1 – Version A

Exponential Functions; Logarithmic functions; Rules of Logarithm

Section 4.1 Exponential Functions

1. Let $f(x) = \left(\frac{1}{4}\right)^x$. Evaluate $f(2), f(0), f(-1). f(\frac{5}{2}), f(-\frac{3}{2})$
2. Simplify each expression using the rules of exponents.
   a) $3^{\sqrt{8}} \cdot 3^{\sqrt{2}}$
   b) $\left(a^{\sqrt{8}}\right)^{\sqrt{2}}$
3. Sketch the graph of the function $f(x) = 2^x$.
4. Sketch the graph of the function $f(x) = \left(\frac{2}{3}\right)^x$.
5. Graph of an exponential function contains the points $(0, 1)$ and $(2, 49)$. Find the function.
6. Graph of an exponential function contains the points $(-2, 16)$ and $(3, \frac{1}{2})$. Find the function.
7. Find the amount that will be in the bank account if \$10,000 is deposited at an annual simple interest rate of 7.5% for two years.
8. Find the amount that will be in the bank account if \$8,000 is deposited at an annual interest rate of 7.5% compounded annually for five years.
9. If \$5,000 id deposited in a bank that pays 6.5% annual interest rate, find the future value A after one year if the interest is compounded
   a) Annually.
   b) Semiannually.
   c) Monthly.
10. John has \$9,000 to invest and he needs \$20,000 at the end of eight years. If the interest is compounded monthly, find the rate r needed.
11. How much will be in the bank at the end of eight years if \$9,000 is invested at 6% rate compounded continuously?
12. Use transformations of the graph of $f(x) = e^x$ to sketch the graph of $g(x) = -e^{x-1} - 2$.
13. In the year 2000, the human population of the world was approximately 6.08 billion. Assume that the annual rate of growth from 1990 onwards at 1.6%. Using the exponential growth model, estimate the population of the world in the year 2030 and in the year 1990.
14. You buy a fishing boat for \$22,000. Your boat depreciates exponentially at the annual rate of 18%. Find the value of the boat at the end of 6 years.

Section 4.2 Logarithmic functions

15. Write each exponential equation in logarithmic form.
   a) $2^{10} = 1024$
   b) $9^{-\frac{1}{2}} = \frac{1}{3}$
   c) $a^{-2} = 7$
16. Write each logarithmic equation in exponential form.
   a) $\log_2 64 = 6$
   b) $\log_u v = w$

17. Evaluate.

    a) $\log_3 9$
    b) $\log_9 \frac{1}{3}$
    c) $\log_{\frac{1}{2}} 32$

18. Evaluate.

    a) $\log_5 1$
    b) $\log_3 3^5$
    c) $7^{\log_7 5}$

19. Find the domain of $\log_{10} \sqrt{1-x}$
20. Sketch the graph of $y = \log_2 x$. Use transformations of the graph of $f(x) = \log_2 x$ to sketch the graph of $y = -\log_2(x-3)$.
21. Evaluate

    a) $\ln e^4$
    b) $\ln \frac{1}{e^{2.5}} \quad \ln 3$

22. How long will it take to triple your money if it earns 6.5% compounded continuously?
23. At what rate of return, compounded continuously, would your money triple in 5 years?
24. The local McDonald's franchise has discovered that when coffee is poured from a coffeemaker whose contents are $180°\text{F}$ into a non- insulated pot, after 1 minute the coffee cools to $165°\text{F}$ if the room temperature is $72°\text{F}$. How long should the employee wait before pouring the coffee from the non-insulated pot into cups to deliver it to customers at $120°\text{F}$?

Section 4.3 Rules of Logarithm

25. Given that $\log_5 y = 2$ and $\log_5 z = 3$. Evaluate

    a) $\log_5 \frac{y}{z}$
    b) $\log_5(y^2 z^3)$

26. Write each expression in expanded form:

    a) $\ln \frac{2x-1}{x+4}$
    b) $\log \sqrt{\frac{4xy}{z}}$

27. Write the expression in condensed form: $\frac{1}{2}[\log(x+1) + \log(x-1)]$
28. Evaluate $\log_5 13$ by changing to (a) common logarithm (b) natural logarithm
29. Find an equation of the form $y = c + b \log x$ whose graph contains the points $(2,3)$ and $(4,-5)$.
30. In an experiment, 100 grams of radioactive element strontium-90 decayed to 66 grams in 15 years. Find its half-life.
31. In 1960, a group of specialists from the British Museum investigated whether a piece of art containing organic material found from Tutankhamun's tomb had been made during his reign or whether it belonged to an earlier period. We know that King Tut died in 1346 B.C. and ruled Egypt for ten years. What percent of the amount of carbon-14 originally contained in the object should be present in 1960 if the object was made 194 years before King Tut's death?