

Section P.5: Rational Expressions

Key Topics: rational expressions, simplified form, operations, LCD, complex fractions

We use the same language to describe rational expressions that we use to describe rational numbers. For $\frac{x^2 + 2x - 3}{x^2 + 5x + 6}$, we call $x^2 + 2x - 3$ the _____ and $x^2 + 5x + 6$ the _____.

The _____ of a rational expression is the set of all real numbers _____ those that result in a _____.

Simplifying Rational Expression

To simplify a rational expression, we

1. _____ its _____ and _____.
2. Divide the numerator _____ denominator by any _____.

Multiplication and Division

For rational expressions $\frac{A}{B}$ and $\frac{C}{D}$,

$$\frac{A}{B} \cdot \frac{C}{D} = \underline{\hspace{2cm}} \quad \text{and} \quad \frac{\frac{A}{B}}{\frac{C}{D}} = \underline{\hspace{2cm}} = \frac{A}{B} \cdot \frac{D}{C} = \underline{\hspace{2cm}}$$

if _____, _____. if _____, _____, _____.

Addition and Subtraction of Rational Expressions

For rational expressions $\frac{A}{D}$ and $\frac{C}{D}$ (_____ denominator _____),

$$\frac{A}{D} + \frac{C}{D} = \underline{\hspace{2cm}} \quad \text{and} \quad \frac{A}{D} - \frac{C}{D} = \underline{\hspace{2cm}}.$$

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To Find the LCD for Rational Expressions

1. _____ denominator polynomial _____.
2. Form a product of the different _____ factors of each polynomial. (Each distinct factor is used once.)
3. Attach to each factor in this product the _____ exponent that appears on this factor in ___ of the factored denominators.

Procedure for Adding or Subtracting Rational Expressions with Different Denominators

- Step 1** Find the _____.
- Step 2** Using $\frac{A}{B} = \frac{AC}{BC}$, write _____ rational expression as a rational expression with the _____ as the _____.
- Step 3** Following the _____ of operations, add or subtract _____, keeping the _____ as the denominator.
- Step 4** _____.

Procedures for Simplifying Complex Fractions

Method 1 Perform the operations indicated in ____ the numerator and denominator of the complex fraction. Then ____ the resulting ____ by the _____.

Method 2 ____ the numerator ____ the denominator of the ____ fraction by the ____ of ____ rational expressions that appear in ____ the numerator or the denominator. ____ the result.

Simplify $\frac{6x^3 + 9x^2 - 27x}{12x^4 + 54x^3 - 108x^2}$ by each Method.