## Section P.6: Rational Exponents and Radicals

**Key Topics:** square root, simplification, rules for exponents

**Principal Square Root**

$\sqrt{a} = b$ means (1) ________ and (2) ______.

$b$ is called **the** ________ **square root** of $a$.

If $\sqrt{a} = b$, then $a = b^2 \geq 0$. Consequently, the symbol $\sqrt{a}$ denotes a real number ___________________ when ___________________. The number or expression under the radical sign is called the ___________________ and the radicand together with the radical sign is called a ___________________.

Note that, $a \geq 0$ ___________________ $\sqrt{a}$ and $-\sqrt{a}$ are square roots of $a$.

**Square Root Property**

For $a \geq 0$, if $x^2 = a$ then __________.

For any real number $x$,

$$\sqrt{x^2} = \_\_\_\_.$$

For any nonnegative real number $x$,

$$(\sqrt{x})^2 = \_\_\_.$$

**Product and Quotient Properties of Square Roots**

$\sqrt{ab} =$ ________ $a$ __ $0$, $b$ __ $0$

$\sqrt{\frac{a}{b}} =$ ______ $a$ __ $0$, $b$ __ $0$

### The Principal *n*th Root of a Real Number

1. If $a$ is positive $(a > 0)$, then $\sqrt[n]{a} =$ _ provided that $b^n =$ _ and ______.
2. If $a$ is negative $(a < 0)$ and $n$ is odd, then $\sqrt[n]{a} =$ _ provided that ______.
3. If $a$ is negative $(a < 0)$ and $n$ is even, then $\sqrt[n]{a}$ is __ a real number.
4. If $a = 0$, then $\sqrt[n]{a} =$ _.

### Simplifying $\sqrt[n]{a^n}$

If $n$ is ___, then $\sqrt[n]{a^n} =$ __.

If $n$ is ____, then $\sqrt[n]{a^n} =$ ___.

### Rules for Radicals

If $a$ and $b$ are real numbers and all indicated roots are defined, then

$\sqrt[n]{ab} =$ ________, ________ **rule**

$\sqrt[n]{\frac{a}{b}} =$ ____, ________ **rule**

and $\sqrt[m]{\sqrt[n]{a}} =$ ____, ______ **rule**

### The Exponent $\frac{1}{n}$

For any real number $a$ and any integer ______,

$$a^{1/n} = \_\_\_\_.$$

When $n$ is even and $a < 0$, $\sqrt[n]{a}$ and $a^{1/n}$ are ___ real numbers.
Notice that the denominator $n$ of the rational exponent is the _____ of the ______.

## The Exponent $\frac{1}{n}$ and Radicals

If $a$ is any real number and $n$ is any __________________, then

| | ***n*** ____ | ***n*** ____ |
|---|---|---|
| If $a > 0$, | $\sqrt[n]{a} =$ ___ is _______. | $\sqrt[n]{a} =$ ___ is _______. |
| If $a < 0$, | $\sqrt[n]{a} =$ ___ is ___ a real number. | $\sqrt[n]{a} =$ ___ is _______ . |
| If $a = 0$, | $\sqrt[n]{0} = 0^{1/n} =$ __. | $\sqrt[n]{0} = 0^{1/n} =$ __. |

## Rational Exponents

$$a^{m/n} = \_\_\_\_\_\_\_ = \_\_\_\_\_,$$

provided that $m$ and $n$ are _______ with no common factors, _____, and $\sqrt[n]{a}$ is a ____ number.

## Properties of Rational Exponents

If $r$ and $s$ are _______ numbers and $a$ and $b$ are ___ numbers, then

$$a^r \cdot a^s = \_\_\_\_\_ \qquad (a^r)^s = \_\_\_ \qquad (ab)^r = \_\_\_\_\_$$

$$\frac{a^r}{a^s} = \_\_\_\_\_ \qquad \left(\frac{a}{b}\right)^r = \_\_\_\_\_ \qquad a^{-r} = \_\_\_\_\_ \qquad \left(\frac{a}{b}\right)^{-r} = \_\_\_\_\_$$

provided that all of the expressions used are ______.