Section 1.4: Complex Numbers: Quadratic Equations with Complex Solutions

Key Topics: complex numbers, square root of negative number, operations of complex numbers, conjugates, discriminant

Definition of *i*

The square root of -1 is called i.

$i =$ _____ so that $i^2 =$ ___.

The number i is called the ______________.

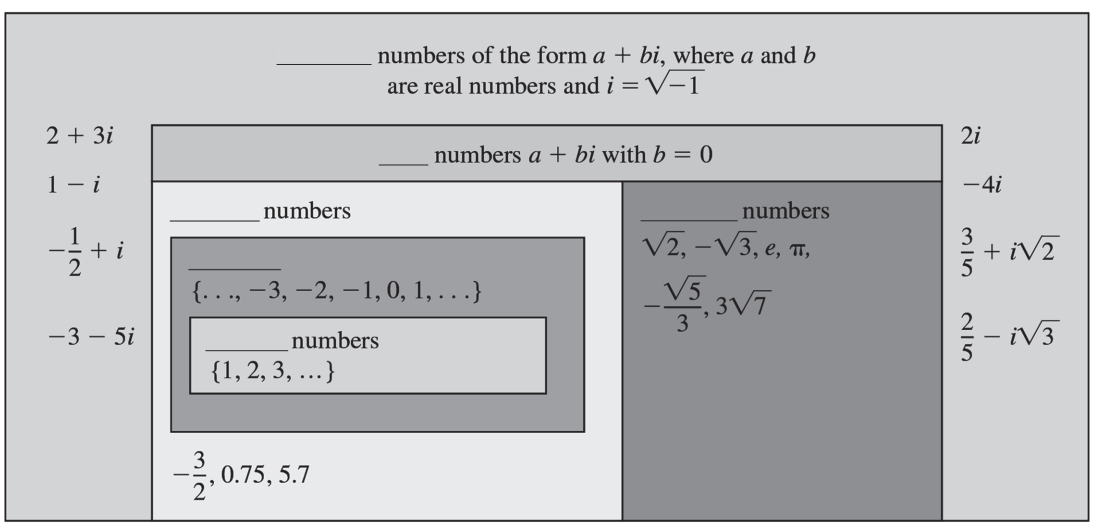
Complex Numbers

A ________ number z is a number of the form

$z =$ ______ ,

where a and b are _____________ and $i =$ _____, (so $i^2 = -1$).

The number a is called the ____ **part** of z, and the number b is called the _________ **part** of z.



Square Root of a Negative Number

For any positive number b,

$$\sqrt{-b} = \_\_\_\_\_\_ = \_\_\_\_.$$

Equality of Complex Numbers

Two complex numbers $z = a + bi$ and $w = c + di$ are _____ if and only if

$$a = \_\_ \quad \text{and} \quad b = \_\_.$$

Addition and Subtraction of Complex Numbers

For real numbers a, b, c, and d, let $z = a + bi$ and $w = c + di$.

Sum: $z + w = (a + bi) + (c + di) =$ __________________

Difference: $z - w = (a + bi) - (c + di) =$ __________________

Multiplying Complex Numbers

For all real numbers a, b, c, and d,

$$(a + bi)(c + di) = \_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_\_.$$

\

Conjugate of a Complex Number

If $z = a + bi$, then the ________ (or complex conjugate) of z is denoted by $\bar{z}$ and defined by $\bar{z} = \overline{a + bi} =$ ______.

Complex Conjugate Product Theorem

If $z = a + bi$, then

$$z\bar{z} = \_\_\_\_\_\_\_.$$

Dividing Complex Numbers

To write the quotient of two complex numbers w and $z\,(z \neq 0)$, write

$\frac{w}{z} =$ ____ Multiply numerator and denominator by__.

and then write the right side in ____________.

Discriminant	Description of Solutions
$b^2 - 4ac$ __ 0	There are two unequal real solutions.
$b^2 - 4ac$ __ 0	There is one real solution.
$b^2 - 4ac$ __ 0	There are two nonreal complex solutions, and they are conjugates.

Solve each equation.

$9x^2 + 25 = 0$

$2x^2 - 20x + 48 = -8$