Section 2.5: Properties of Functions

Key Topics: increasing, decreasing, constant, even, odd, and piecewise functions, average rate of change, difference quotient

Increasing, Decreasing, and Constant Functions

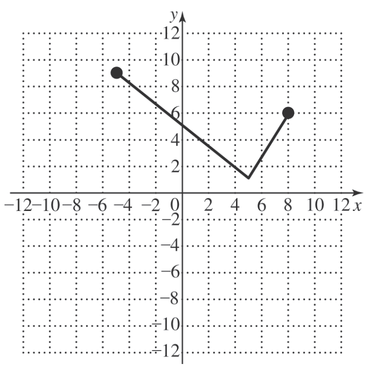
Let f be a function and let x_1 and x_2 be any two numbers in an open interval (a, b) contained in the domain of f. See Figure 1.61. The symbols a and b may represent real numbers, $-\infty$, or ∞. Then

(i) f is called an _________________ on (a, b) if $x_1 < x_2$ implies $f(x_1) < f(x_2)$.

(ii) f is called a _________________ on (a, b) if $x_1 < x_2$ implies $f(x_1) > f(x_2)$.

(iii) f is ________ on (a, b) if $x_1 < x_2$ implies $f(x_1) = f(x_2)$.

From the graph of the function g, find the interval over which g is increasing.



(a) $(3,8)$ (b) $(-6,3)$

(c) $(1,6)$ (d) $(1,9)$

Find the interval over which g is decreasing.

Relative Maximum and Relative Minimum

If a is in the domain of a function f, we say that the value $f(a)$ is a ____________ ________ if there is an interval (x_1, x_2) containing a such that

$$f(a) \geq f(x) \text{ for every } x \text{ in the interval } (x_1, x_2).$$

We say that the value $f(a)$ is a ____________________ if there is an interval (x_1, x_2) containing a such that

$$f(a) \leq f(x) \text{ for every } x \text{ in the interval } (x_1, x_2).$$

Even-Odd Functions

A function f is called an ____________ if, for each x in the domain of f, $-x$ is also in the domain of f and

$$f(-x) = f(x).$$

The graph of an even function is symmetric with respect to the y-axis.

A function f is an ____________ if for each x in the domain of f, $-x$ is also in the domain of f and

$$f(-x) = -f(x).$$

The graph of an odd function is symmetric with respect to the origin.

The Average Rate of Change of a Function

Let $(a, f(a))$ and $(b, f(b))$ be two points on the graph of a function f. Then the **average rate of change** of $f(x)$ as x changes from a to b is defined by

ARC of f from a to b = __________ = ____ = ___________, ______.

Difference Quotient

For a function f, the **difference quotient** is

______________, ______.

Find and simplify the difference quotient for $f(x) = x^2 + 4x - 5$.