Section 2.7: Transformation of Functions

Key Topics: vertical and horizontal shifts, reflections, stretching, compressing

VERTICAL SHIFT

Let $d > 0$. The graph of $g(x) = f(x) + d$ is the graph of $y = f(x)$ shifted d units __, and the graph of $h(x) = f(x) - d$ is the graph of $y = f(x)$ shifted d units _____.

If (x, y) is a point on the graph of $y = f(x)$, then the corresponding point $(x, y + d)$ is on the graph of $y = f(x) + d$ and the point $(x, y - d)$ is on the graph of $y = f(x) - d$.

HORIZONTAL SHIFTS

Let $c > 0$. The graph of $y = f(x - c)$ is the graph of $y = f(x)$ shifted c units to the ____. The graph of $y = f(x + c)$ is the graph of $y = f(x)$ shifted c units to the ___.

If (x, y) is a point on the graph of f, then the corresponding point $(x + c, y)$ is on the graph of $y = f(x - c)$, and the point $(x - c, y)$ is on the graph of $y = f(x + c)$.

Graphing Combined Vertical and Horizontal Shifts

OBJECTIVE

Sketch the graph of $g(x) = f(x \pm c) \pm d$, where f is a function whose graph is known.

Step 1 ____

Step 2 ____

Step 3 ____

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Step 4 ____

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REFLECTION ABOUT THE x-AXIS

The graph of $g(x) = -f(x)$ is a reflection of the graph of $y = f(x)$ about the x-axis. If a point (x, y) is on the graph of f, then the point ______ is on the graph of g.

REFLECTION ABOUT THE y-AXIS

The graph of $g(x) = f(-x)$ is a reflection of the graph of $y = f(x)$ about the y-axis. If a point (x, y) is on the graph of f, then the point ______ is on the graph of g.

VERTICAL STRETCHING OR COMPRESSING

The graph of $g(x) = af(x)$ is obtained from the graph of $y = f(x)$ by multiplying the y-coordinate of each point on the graph of $y = f(x)$ by a and leaving the x-coordinate unchanged. The result is as follows:

1. A ______________ away from the x-axis if $a > 1$

2. A ___________________ toward the x-axis if $0 < a < 1$.

If $a < 0$, first graph $y = |a|f(x)$ by stretching or compressing the graph of $y = f(x)$ vertically. Then reflect the resulting graph about the x-axis.

So, if (x, y) is a point on the graph of f, then the point (x, ay) is on the graph of g.

HORIZONTAL STRETCHING OR COMPRESSING

Let $b > 0$. The graph of $g(x) = f(bx)$ is obtained from the graph of $y = f(x)$ by multiplying the x-coordinate of each point on the graph of $y = f(x)$ by $\frac{1}{b}$ and leaving the y-coordinate unchanged. The result is as follows:

1. A ________________ away from the y-axis if $0 < b < 1$.

2. A _____________________ toward the y-axis if $b > 1$.

If $b < 0$, first graph $f(|b|x)$ by stretching or compressing the graph of $y = f(x)$ horizontally. Then reflect the graph of $y = f(|b|x)$ about the y-axis.

If (x, y) is a point on the graph of $y = f(x)$, then the point $\left(\frac{1}{b}x, y\right)$ is on the graph of g.

Multiple Transformations in Sequence

When graphing requires more than one transformation of a basic function, it is helpful to perform transformations in the following order:

1. ______________ **2.** ______________ **3.** ______________ **4.** ______________

Sketch the graph of the function $f(x) = 9 - 3(x+2)^2$.