Section 2.8: Combining Functions; Composite Functions

Key Topics: sum, difference, product, quotient, and composite functions

Sum, Difference, Product, and Quotient of Functions

Let f and g be two functions. The **sum** $f + g$, the **difference** $f - g$, the **product** fg, and the **quotient** $\frac{f}{g}$ are functions defined as follows:

- Sum ______________________________
- Difference ______________________________
- Product ______________________________
- Quotient ______________________________

The **domain** of each of the new functions consists of those values of x that are common to the domains of f and g, except that for $\frac{f}{g}$, all x for which $g(x) = 0$ must also be excluded.

Composition of Functions

If f and g are two functions, the composition of the function f with the function g is written as $f \circ g$ and is defined by the equation

$$(f \circ g)(x) = \_\_\_\_\_\_$$

We read $(f \circ g)(x)$ as "f composed with g of x" and $f(g(x))$ as "f of g of x". The domain of $f \circ g$ consists of those values x in the domain of g for which $g(x)$ is in the domain of f.

Let $f(x) = 2x^2 + 7$ and $g(x) = 3x - 1$. Find the composite function $(f \circ g)(x)$.

Decomposing a Function

OBJECTIVE

Write a function H as a composite of simpler functions f and g so that $H = f \circ g$.

Step 1 Define $g(x)$ as any expression in the defining equation for H.

Step 2 To get $f(x)$ from the defining equation for H, (1) replace the letter H with f and (2) replace the expression chosen for $g(x)$ with x.

Step 3 Now we have

$$H(x) = \_\_\_\_\_\_\_\_\_ \\ = \_\_\_\_\_\_\_\_\_$$

Write $H(x) = (x+3)^2 - 2(x+3) + 1$ as $f \circ g$ for some functions f and g.

Section 2.9: Inverse Functions

Key Topics: one-to-one functions, horizontal line test, inverse function

One-to-One Function

- If different elements in the domain of *f* are assigned to different values, then the function *f* is __________. That is, if $x_1 \neq x_2$, then $f(x_1) \neq f(x_2)$.
- If there are two distinct elements in the domain of *f* that are assigned the same value, then the function *f* is ____________.

Horizontal-Line Test

- If no horizontal line intersects the graph of the function more than once, then the function is __________.
- If some horizontal line intersects the graph of the function more than once, then the function is ____________.

Inverse Function

Let f represent a one-to-one function. Then if y is in the range of f, there is only one value of x in the domain of f such that $f(x) = y$. We define the inverse of f, called the **inverse function of *f***, denoted f^{-1}, by $f^{-1}(y) = x$ if and only if ________.

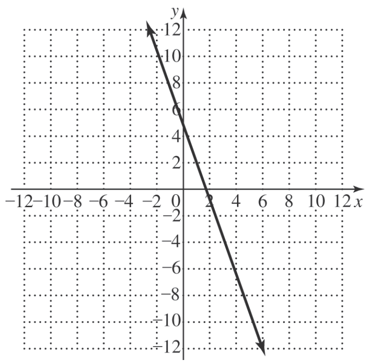
Domain of $f =$ _______________ and Range of $f =$ _______________

Assume that *f* is a one-to-one function. If $f(-3) = 11$, find $f^{-1}(11)$

SYMMETRY PROPERTY OF THE GRAPHS OF f AND f^{-1}

The graph of a one-to-one function f and the graph of f^{-1} are symmetric with respect to the line ______.

The graph of a function f is shown. Sketch the graph of f^{-1}.



Find the inverse of the one-to-one function $f(x) = 2x - 7$.