Chapter 3: Polynomial and Rational Functions

Section 3.1: Quadratic Functions

Key Topics: quadratic function, standard form, graphing techniques, vertex, maximum, minimum

Quadratic Function

A function of the form

$$f(x) = \_\_\_\_\_\_\_\_\_\_\_\_,$$

where a, b, and c are real numbers with $a \neq 0$, is called a **quadratic function**.

The Standard Form of a Quadratic Function

The quadratic function

$$f(x) = a(x - h)^2 + k, \ a \neq 0$$

is in _____________. The graph of f is a parabola with ______ (h, k).
The parabola is symmetric with respect to the line $x = h$, called the ____ of the parabola. If $a > 0$, the parabola opens up, and if $a < 0$, the parabola opens down. If $a > 0$, k is the _________________, and if $a < 0$, k is the _________________.

Graphing a Quadratic Function in Standard Form

OBJECTIVE

Sketch the graph of $f(x) = a(x - h)^2 + k$.

Step 1 The graph is a parabola because it has the form $f(x) = a(x - h)^2 + k$. Identify _, _, and _.

Step 2 Determine how the parabola opens. If $a > 0$, the parabola opens __. If $a < 0$, it opens ____.

Step 3 Find the vertex (h, k). If $a > 0$ (or $a < 0$), the function f has a minimum (or a maximum) value k at $x = h$.

Step 4 Find the x-intercepts (if any). Set $f(x) = 0$ and solve the equation _________________ for x. If the solutions are real numbers, they are the x-intercepts. If not, the parabola lies above the x-axis (when $a > 0$) or below the x-axis (when $a < 0$).

Step 5 Find the y-intercept. Replace x with 0. Then ______________ is the y-intercept.

Step 6 Sketch the graph. Plot the points found in Steps 3–5 and join them to form a parabola. Show the axis $x = h$ of the parabola by drawing a dashed vertical line.

If there are no x-intercepts, draw the half of the parabola that passes through the vertex and a second point, such as the y-intercept. Then use the axis of symmetry to draw the other half.

Finding the Vertex of $f(x) = ax^2 + bx + c, a \neq 0$

To find the vertex (h, k) of $f(x) = ax^2 + bx + c$,

1. Find the x-coordinate $h =$_____ of the vertex.

2. Calculate $k = f(\_\_\_\_)$ to find its y-coordinate.

3. If $a > 0$, then __ is the _________ value of f.

4. If $a < 0$, then __ is the _________ value of f.

Graph $f(x) = x^2 - 4x - 5$

i. Parabola opens ____________________________________.

ii. Vertex ____________________________________.

iii. Axis of symmetry ____________________________________.

iv. x-intercepts? ____________________________________.

v. y-intercept ____________________________________.

FINDING THE RANGE OF $f(x) = ax^2 + bx + c$ HAVING VERTEX (h, k)

1. If $a > 0$, the range of $f(x) = ax^2 + bx + c$ is ______.

2. If $a < 0$, the range of $f(x) = ax^2 + bx + c$ is _______.

Summary of Main Facts

The quadratic equation $f(x) = ax^2 + bx + c = 0, a \neq 0$ can be written in standard form $f(x) =$ ______________ by______________________.

The vertex is_____ ; $h =$ _____ and $k =$______.

The_______ line $x =$ __ is the ________________.
The graph opens___ if $a > 0$ and _____ if $a < 0$.
The graph is_____ than that of $y = x^2$ if $|a|$ ___1 and ________ if $|a|$___1.
The y-intercept is___.
The x-intercepts are the solutions of the equation ___________________.

a. If $b^2 - 4ac$ ___0, the x-intercepts are $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

b. If $b^2 - 4ac$ ___0, there is ________ x-intercept, $x = -\dfrac{b}{2a}$.

c. If $b^2 - 4ac$ ___0, there is___ x-intercept.

Graph the function $f(x) = x^2 - 6x + 7$.

Determine:

Vertex
Axis of symmetry
Intercept(s)