Section 3.4: The Real Zeros of a Polynomial Function

Key Topics: Rational Zeros Theorem, Conjugate Pairs Theorem, Descarte's Rule of Signs, Rules for Bounds on the Zeros

Rational Zeros Theorem

If $F(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_2 x^2 + a_1 x + a_0$ is a polynomial function with integer coefficients $(a_n \neq 0, a_0 \neq 0)$ and $\frac{p}{q}$ is a rational number in lowest terms that is a zero of $F(x)$, then

1. __ is a factor of the constant term a_0.

2. __ is a factor of the leading coefficient a_n.

Conjugate Pairs Theorem for Irrational Zeros

If $P(x)$ is a polynomial with _____ coefficients and $a + b\sqrt{c}$ (where a, b, and c are integers with _____ is an _____ zero of $P(x)$, then _____ is also an irrational zero of $P(x)$.

DESCARTES'S RULE OF SIGNS

Let $F(x)$ be a polynomial function with real coefficients and with nonzero terms written in _____ powers.

1. The number of _____ zeros of F is equal to the number of variations of sign of $F(x)$ or is less than that number by an even integer.

2. The number of _____ zeros of F is equal to the number of variations of sign of $F(-x)$ or is less than that number by an even integer.

When using Descartes's Rule, a zero of _____ m should be counted as m zeros.

Rules for Bounds

Let $F(x)$ be a polynomial function with real coefficients and a positive leading coefficient. Suppose $F(x)$ is synthetically divided by $x - k$.

1. If $k > 0$ and each number in the last row is zero or positive, then k is an _____ bound on the zeros of $F(x)$.
2. If for any k, the numbers in the last row alternate in sign, then k is a _____bound on the zeros of $F(x)$.

Zeros in the last row can be regarded as positive or negative.

Find all rational zeros of the polynomial $P(x) = x^4 - x^3 - 8x^2 - 4x - 48$.