Section 3.5: The Complex Zeros of a Polynomial Function

Key Topics: Fundamental Theorem of Algebra, Factorization Theorem for Polynomials, Number of Zeros Theorem, Conjugate Pairs Theorem, Odd-Degree Polynomials, Factorization Theorem for a Polynomial with Real Coefficients

FUNDAMENTAL THEOREM OF ALGEBRA

Every polynomial

$$P(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots + a_1 x + a_0 \quad (n \geq 1, a_n \neq 0)$$

with complex coefficients $a_n, a_{n-1}, \ldots, a_1, a_0$ has ______________________.

Factorization Theorem for Polynomials

If $P(x)$ is a complex polynomial of degree $n \geq 1$ with leading coefficient a, it can be factored into n (not necessarily distinct) linear factors of the form

$$P(x) = \underline{\hspace{6cm}},$$

where $a, r_1, r_2, \ldots, r_n$ are complex numbers.

NUMBER OF ZEROS THEOREM

Any polynomial of degree n has ______ n zeros, provided a zero of multiplicity k is counted k times.

CONJUGATE PAIRS THEOREM

If $P(x)$ is a polynomial function whose coefficients are real numbers and if $z = a + bi$ is a zero of P, then its conjugate, __________, is also a zero of P.

Odd-Degree Polynomials Have a Real Zero

Any polynomial $P(x)$ of odd degree with real coefficients must have ____________ _________.

FACTORIZATION THEOREM FOR A POLYNOMIAL WITH REAL COEFFICIENTS

Every polynomial with real coefficients can be ________ factored over the real numbers as a product of linear factors and/or irreducible quadratic factors.

Find all zeros of the polynomial $P(x) = x^4 - x^3 - 8x^2 - 4x - 48$.