Section 3.6: Rational Functions

Key Topics: rational functions, vertical asymptotes, horizontal asymptotes, graphing techniques

Rational Function

A function f that can be expressed in the form

$$f(x) = \_\_\_\_\_\_$$

where the numerator $N(x)$ and the denominator $D(x)$ are polynomials and $D(x)$ is not the zero polynomial is called a **rational function**. The domain of f consists of all real numbers for which $D(x) \neq 0$.

Find the domain of the rational function $f(x) = \dfrac{x-1}{x^2+x-6}$.

Vertical Asymptote

The line $x = a$ is called a **vertical asymptote** of the graph of a function f if ____.

Horizontal Asymptote

The line $y = k$ is a **horizontal asymptote** of the graph of a function f if $f(x) \to k$ ______________________________.

LOCATING VERTICAL ASYMPTOTES OF RATIONAL FUNCTIONS

If $f(x) = \dfrac{N(x)}{D(x)}$ is a rational function, where $N(x)$ and $D(x)$ ____________________ _____ and a is a real zero of $D(x)$, then the line with equation ______ is a vertical asymptote of the graph of f.

Find all vertical asymptotes of the rational function $f(x)=\dfrac{x-1}{x^2-25}$.

RULES FOR LOCATING HORIZONTAL ASYMPTOTES

Let f be a rational function given by

$$f(x)=\frac{N(x)}{D(x)}=\frac{a_n x^n+a_{n-1}x^{n-1}+\cdots+a_2x^2+a_1x+a_0}{b_m x^m+b_{m-1}x^{m-1}+\cdots+b_2x^2+b_1x+b_0},\ a_n\neq 0,\ b_m\neq 0$$

To find whether the graph of f has one horizontal asymptote or no horizontal asymptote, we compare the degree of the numerator, n, with that of the denominator, m:

1. If $n < m$, then the ______________ is the horizontal asymptote.
2. If $n = m$, then the line with equation ________ is the horizontal asymptote.
3. If $n > m$, then the graph of f has ___ horizontal asymptote.

Find the horizontal asymptote of the graph of the rational function $f(x)=\dfrac{6-5x}{3x-7}$.

Determine the oblique asymptote for $f(x)=\dfrac{x^3-3x}{x^2+1}$

Sketch the graph of $f(x)=\dfrac{x^2-9}{x+2}$