

Section 3.7: Variation**Key Topics:** variation relationships—direct, inverse, joint, combined**Direct Variation**

A quantity y is said to _____ as the quantity x , or y is _____ to x , if there is a constant k such that $y = kx$. This constant k is called the _____ or the _____.

Suppose y is directly proportional to x , and $y = 40$ when $x = 10$. Find y if $x = 16$.

Direct Variation with Powers

“A quantity y varies directly as the n th power of x ” means

$$y = _,$$

where k is a nonzero constant and $n > 0$.

Inverse Variation

A quantity y _____ as the quantity x , or y is _____ to x , if

$$y = \frac{k}{x}$$

where k is a nonzero constant.

Suppose y varies inversely as x , and $y = 12$ when $x = 6$. Find y if $x = 8$.

Inverse Variation with Powers

A quantity y varies inversely as the n th power of x means that $y = _$, where k is a nonzero constant and $n > 0$.

Joint and Combined Variation

The expression z varies jointly as x and y means that $z = kxy$ for some nonzero constant k . Similarly, if n and m are positive numbers, then the expression z varies jointly as the n th power of x and m th power of y means that $z = kx^n y^m$ for some nonzero constant k .

SOLVING VARIATION PROBLEMS

Step 1 Write the equation that models the problem.

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|--|------------------------|
| a. y varies directly with x . | $y = kx$ |
| b. y varies with the n th power of x . | $y = kx^n$ |
| c. y varies inversely with x . | $y = \frac{k}{x}$ |
| d. y varies inversely with the n th power of x . | $y = \frac{k}{x^n}$ |
| e. z varies jointly with the n th power of x and the m th power of y . | $z = kx^n y^m$ |
| f. z varies directly with the n th power of x and inversely with the m th power of y . | $z = \frac{kx^n}{y^m}$ |

Step 2 Substitute the given values in the equation in Step 1 and solve for k .

Step 3 Rewrite the equation in Step 1 with the value of k from Step 2.

Step 4 Use the equation from Step 3 to answer the question posed in the problem.