Chapter 4: Exponential and Logarithmic Functions

Section 4.1: Exponential Functions

Key Topics: exponential functions, rules of exponents, graphing techniques, properties of exponential functions, transformations, simple interest, compound interest, continuous compound interest, natural exponential function, exponential growth and decay

Exponential Function

A function f of the form

$$f(x) = a^x, a > 0, \text{and } a \neq 1,$$

is called an _____. Its domain is $(-\infty, \infty)$.

Rules of Exponents

Let a, b, x, and y be real numbers with $a > 0$ and $b > 0$.

$a^x \cdot a^y =$ ____ $\quad \frac{a^x}{a^y} =$ ____ _____

$a^0 =$ ____ $\quad a^{-x} =$ ____ _____

$(a^x)^y =$ ____ _____

$(ab)^x =$ ____ $\quad \left(\frac{a}{b}\right)^x =$ ____ _____

Graph the exponential function $f(x) = 2^x$.

Graph the exponential function $f(x) = \left(\frac{1}{3}\right)^x$

PROPERTIES OF EXPONENTIAL FUNCTIONS

Let $y = f(x) = a^x, a > 0, a \neq 1$.

1. The domain of f is _____.
2. The range of f is _____: The entire graph lies above the x-axis.
3. Note that $f(x + 1) = a^{x+1} = a \cdot a^x = af(x)$. This means that the y-values change by a _____.
4. For $a > 1$, the _____ is a. Because the y-values _____ by a factor of a for each unit increase in x,
   (i) f is an _____ function, so the graph ____ to the right.
   (ii) as $x \to \infty$, _____.
   (iii) as $x \to -\infty$, _____.
5. For $0 < a < 1$, the _____ is a. Because the y-values _____ by a factor of a for each unit increase in x,
   (i) f is a _____ function, so the graph ____ to the right.
   (ii) as $x \to -\infty$, _____.
   (iii) as $x \to \infty$, _____.
6. Each exponential function f is one-to-one. So,
   (i) if $a^m = a^n$, then _____.
   (ii) f has _____.
7. The graph of f has _____, so it _____ crosses the x-axis. No value of x will cause $f(x) = a^x$ to equal 0.
8. The graph of f is a smooth and continuous curve, and it passes through the points _____.
9. The x-axis $(y = 0)$ is a _____ asymptote for the graph of _____ exponential function of the form $f(x) = a^x$.
10. The graph of $y = a^{-x}$ is the _____ about the _____ of the graph of $y = a^x$.

Transformations on the graph of $y = a^x$

Transformation (in each case $c > 0$)	**Change the graph point (x, y) on $y = a^x$ to**	**Description**
Vertical shift $y = a^x + c$	$(x, \_\_\_\_)$	Shift the graph of $y = a^x$ ___ c units. Horizontal asymptote: _____
$y = a^x - c$	$(x, \_\_\_\_)$	Shift the graph of $y = a^x$ ____ c units. Horizontal asymptote: _____
Horizontal shift $y = a^{x-c}$ $y = a^{x+c}$	$(\_\_\_\_, y)$ $(\_\_\_\_, y)$	Shift the graph of $y = a^x$ to the ____ c units. Shift the graph of $y = a^x$ to the ___ c units.
Reflection $y = -a^x$ $y = a^{-x}$	$(x, \_\_)$ $(\_\_, y)$	Reflect the graph of $y = a^x$ about the _____. Reflect the graph of $y = a^x$ about the _____.
Vertical stretching or compressing $y = ca^x$	$(x, \_\_)$	Vertically _____ the graph of $y = a^x$ if $c > 1$. Vertically _____ the graph of $y = a^x$ if $0 < c < 1$.
Horizontal stretching or compressing $y = a^{cx}$	$\left(\_, y\right)$	Horizontally _____ the graph of $y = a^x$ if $c > 1$. Horizontally _____ the graph of $y = a^x$ if $0 < c < 1$.

Sketch the graph of $f(x) = 3^{x+2} - 4$.

Simple Interest

A fee charged for borrowing a lender's money is the _____, denoted by *I*.

The original, or initial, amount of money borrowed is the _____, denoted by *P*.

The period of time during which the borrower pays back the principal plus the interest is the ____, denoted by *t*.

The _____ is the percent charged for the use of the principal for the given period. The interest rate, denoted by *r*, is expressed as a decimal. Unless stated otherwise, the period is assumed to be one year; that is, *r* is an _____ rate.

The amount of interest computed only on the principal is called _____.

SIMPLE INTEREST FORMULA

The simple interest *I* on a principal *P* at a rate *r* (expressed as a decimal) per year for *t* years is

$$I = \_\_\_.$$

Mykala has deposited $5,000 in a bank for six years at a simple interest rate of 5%. How much money will she receive at the end of the six years?

COMPOUND INTEREST FORMULA

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$A =$ _____

$P =$ _____

$r =$ _____ (_____)

$n =$ number of _____

$t =$ number of ____

CONTINUOUS COMPOUND INTEREST FORMULA

$$A = \_\_\_\_$$

A = amount after t years
P = principal
r = annual interest rate (expressed as a decimal number)
t = number of years

If \$500 is deposited in a bank that pays $1\frac{3}{4}\%$ annual interest, find the future value A after three years if the interest is compounded

i. annually ii. quarterly iii. monthly iv. daily v. continuously

Models for Linear Growth and Decay

_____ growth and decay can be expressed as

$L(t) =$ _____ where

$L(t) =$ _____

$L_0 =$ ____, the _____ amount (the amount at _____)

$k =$ _____ $(k > 0)$ or _____ $(k < 0)$

$t =$ _____

Models for Exponential Growth and Decay

_____ growth and decay can be expressed as

$A(t) =$ _____ where

$A(t) =$ _____

$A_0 =$ ____, the _____ amount (the amount at _____)

$k =$ relative rate of growth _____or decay_____

$t =$ _____

The number of bacteria at the beginning of an experiment was 30 and the bacteria grow at an hourly rate of 1.4 percent. Using the model given by $A(t) = A_0 e^{kt}$, estimate the number of bacteria, rounded to the nearest whole number, after 20 hours.